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ABSTRACT

This collection of activities is organized into two sections. The first, entitled "Manipulatives," suggests materials which may be used to introduce or reinforce mathematical concepts such as: basic arithmetic operations; place value; long division; percents; multiples and common denominators; informal geometry including area, perimeter and volume; and pattern recognition and other problem-solving strategies. The second section, "Games," reflects the authors' conviction that games have a contribution to make in the mathematics classroom, particularly in the areas of basic skills practice, applications, and logic and strategy development. Like the first section, it is organized by grade level. The majority of activities are appropriate for the primary and elementary levels although many include variations suitable for higher grade levels. None of the activities described requires the purchase of commercial materials and both sections are prefaced with articles providing a basic rationale for the use of manipulatives and games which may prove helpful when dealing with skeptical parents or administrators. (MM)

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Manipulative Activities and Games in the Mathematics Classroom

Lee E. Vochko, Editor

The Curriculum Series

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National Education Association
Washington, D.C.

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CONTENTS

PART I Manipulatives

PART II Games

[illegible]

This manipulative guide [illegible] contains activities [illegible] teachers of math [illegible] the United States. [illegible] diagrams and examples [illegible]

Manipulative activities are presented in the first section of the book. They are arranged by suggested grade level beginning with primary. [illegible] introductory activities [illegible] manipulatives and games [illegible] playing games and learning.

All activities have a suggested grade level included but these are only suggestions. [illegible] appropriate for your class. [illegible]

The material chosen is easy and inexpensive to make. [illegible] commercial materials [illegible] Blank playing cards [illegible] supply companies [illegible]

This book is not intended to be an encyclopedia of all math manipulatives [illegible] Rather it is a collection that can be used as a basis for introducing manipulatives [illegible] you and your students have a fun time learning math.

[illegible]

The Editor

Lisa K. [illegible] is a seventh and eighth grade mathematics teacher at the [illegible], Huntington Beach, California.

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GETTING IT ALL TOGETHER WITH MATH LEARNING CENTERS

by Patricia C. Manning

This selection describes learning centers that teachers can make from recycled materials. The specific examples given are aimed at the primary grades but can easily be adapted to higher grade levels. Patricia C. Manning is an Assistant Professor of Elementary Education, Florida Technological University, Orlando.

Have you tried learning centers? Why not begin trying learning centers in your classroom? Whether you are teaching 6-, 16-, or 26-year-olds, learning centers can provide a new and exciting approach to the teaching of math. Not only can math learning centers help *you* to "get it all together," but also they can motivate and help your *students* to "get it all together." Learning centers can be an extra set of hands for you, they can be used to introduce a new skill or concept, to reinforce a skill, and to provide additional practice of a skill. Learning centers can enrich, reinforce, drill, enlighten, relieve boredom, and reduce discipline problems. Learning centers can be a third hand, assisting you when working with large groups, small groups, or individuals. Learning centers can be a teaching tool and are a teaching technique. Have I convinced you? If I have, don't stop now there are some good ideas that follow.

There are many ways to utilize learning centers in teaching math in an elementary or middle school classroom. But before you implement the learning center approach in your classroom, begin collecting odds and ends of materials (bottle caps, stirrers, tongue depressors, egg cartons, old catalogs, menus, pieces of styrofoam, packing materials the list goes on and on). Then begin placing these materials ("Happy Trash") in boxes labeled with the contents. Now you are ready to begin putting activities together from the collection of materials.

Learning centers need not and *should not* be expensive or time consuming for you to make. Older students could aid in making activities for learning centers. Parent volunteers, paraprofessionals, and teachers in a building could develop banks of learning centers to share and cooperatively use.

If you are a primary teacher, here are a few suggestions to get you started in making activities for your centers. Make a clown out of cardboard with a numeral on the hat, then have the student place buttons on the clown's costume accordingly. Or cut the shape of a shirt or blouse out of cardboard and color it, make buttonholes and place the corresponding number of buttons on one side of the blouse or shirt. Make washcloths with numerals on them, the student places the washcloth on a clothesline, and the correct number of clothespins are placed on the washcloth according to the number sewn on it. Activities of this type could be put in individual boxes, labeled (with symbols and words), and placed so that they are easily accessible to teacher and student. A backdrop of cardboard or a sewing cutting board could be placed on a table or floor in the classroom with the boxes of the various activities in front. The student could have the teacher check the activity. A list could be made of each student in the classroom with the activity listed at the top, and when the student has successfully completed the skill, she/he can then cross her/his name of the list. This activity checklist can then be used at a later date for recording information for diagnostic and evaluation purposes.

Learning centers in the intermediate grades could become more detailed as to what is expected of the student as she/he progresses through tasks that are outlined at the center.

Here are some titles for your centers: "The Fact Fair," "Math Path," "Got a Minute?" "Is Math Bugging You?" or "The Times of Your Life." For example title a learning center "The Fact Fair," and the objective for the center is to solve basic addition and subtraction facts. "The Fact Fair" center could have a picture of a clown holding balloons with simple facts written on them. The picture of the clown could be fastened to a backdrop made of cardboard and placed on the floor. In front of the backdrop, place boxes and file folders with different activities. File folders can be laminated or covered with clear Con-Tact paper to preserve the work that has gone into making them. Here again, students, aides, and parents can help.

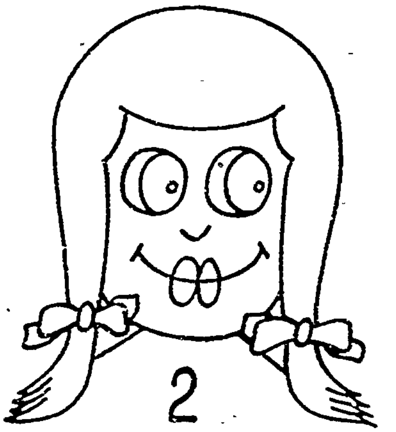
The activities should all be labeled and numbered, and then again a checklist should be at the center for the student to let you know that he/she has finished the activity. Or each student could have a folder with the activities marked in the folder, with either paper or a checklist denoting a finished or unfinished activity.

The activities in your center should represent a multi-level approach in order for all students to be able to manipulate and work with the activities. All the students should have equal time to work the activities, not just when they finish their work on the purple ditto sheet. This approach will begin to do away with the ditto dragon and the ugly purple that have become such a way of life in many classrooms.

As money for consumable items is depleted in our public schools, teachers will have to resort to using their innate creativity and innovativeness that are such a part of every excellent, creative classroom teacher. But words of caution should be expressed before you begin the task of opening up your classroom to learning centers. Go very slowly, try just one academic area at a time—math is the best—and try just one center at a time. You can begin to get it all together utilizing math centers with your children.

The illustrations and descriptions that follow will aid you in your quest for math manipulatives to be used in your math learning centers.

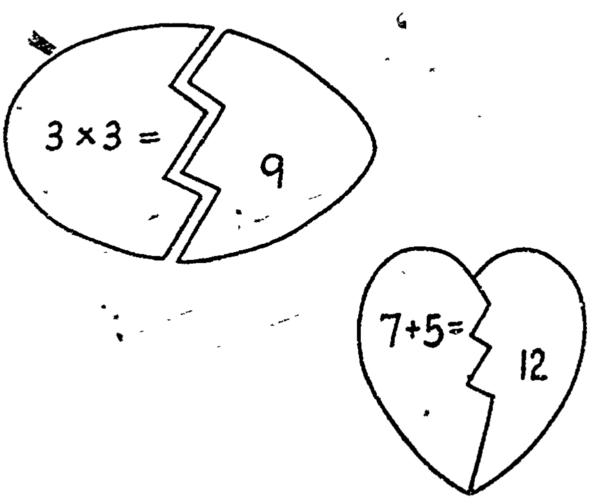
"Betty Barrette"



Materials: Tagboard, pieces of cloth remnants or wallpaper samples, magic markers, glue, small plastic barrettes, colored yarn, scissors.

Procedure: Cut out the shapes of a small girl's head and shoulders. Draw faces on the cut-out heads. On the shoulders of the figures glue remnants or wallpaper to resemble a dress. Glue shoulder length yarn around the face to resemble hair. On the front of the dress write a numeral. The student is to place the correct number of barrettes on the yarn that matches the numeral on the dress

"Broken Heart/Egg Math"



Materials: Construction paper or tagboard, magic markers, clear Con-Tact paper, scissors.

Procedure: Cut out shapes to resemble an egg or a heart. After the shapes have been cut out, outline them in magic marker, and on one half write a problem (for example, an addition fact) and on the other half the correct answer. Cut the shapes in an irregular pattern, so that the student will have to match pieces of the puzzle. Finding the correct match will produce the answer. Puzzles can also be made for subtraction, multiplication, division, and fractions.

"Charlie Cone"

Materials: Construction paper or tagboard, clear Con-Tact paper, scissors, magnetic strip tape, staples, magic markers.

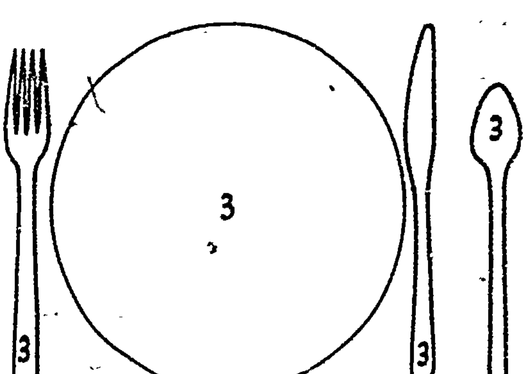
Procedure: Cut out of construction paper or tagboard shapes that resemble an ice cream cone. On the top of the cone draw a face. Place a small piece of the magnetic tape on the top and bottom of the face. Cut out small hats and bow ties, and write addition facts on the hats and the answers on the bow ties. Place a staple on each hat and each bow tie. This staple will attract the hat and bow tie to the magnetic tape on the cone. Students are to correctly match the facts. This activity can be made for all the basic facts.

"Sammy Seal"

Materials: Tagboard, magnetic strip tape, magic markers, hole punch, scissors.

Procedure: Cut out shapes to resemble a seal balancing balls. On the seal's nose and on the back of each ball place a small piece of magnetic tape. Write a numeral on the body of the seal and on each ball punch the corresponding number of holes. The student is to match the number of holes in the ball with the correct numeral on the seal.

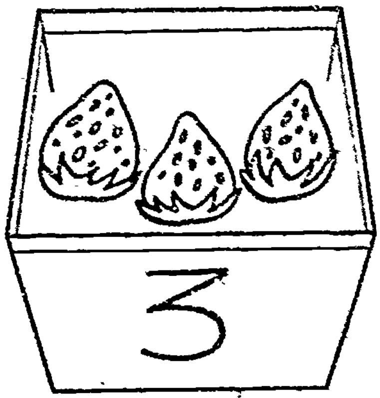
"Set The Table"



Materials: Ten small, white paper plates; ten plastic spoons, forks, and knives; large, self-closing plastic bag; shoe box; magic markers.

Procedure: Write numerals on the individual paper plates and utensils. The students are to set the table matching the numerals. All the materials can be stored in the plastic bag and placed in a shoe box.

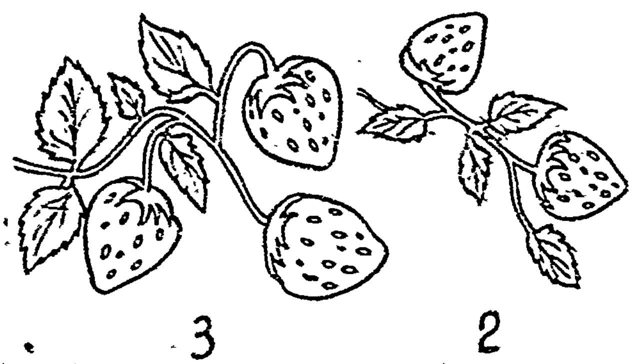
"Strawberry Picking"



Materials. Ten empty strawberry boxes, small pieces of construction paper, glue, small stones, red and green paint, shoe box.

Procedure. Write the numerals 1 to 10 on small pieces of construction paper that will fit on the front of each strawberry box. Glue them to the strawberry boxes. Paint the small stones to resemble strawberries. The student is to place the correct number of strawberries in each basket. Store the materials in a shoe box.

"Strawberry Vines"



Materials: Box or file folder, magic markers, small stones, small plastic bag, red and green paint

Procedure. Paint the small stones to resemble strawberries. Inside the bottom of a box or across the inside of a file folder, draw a vine with leaves. At the end of the vine write a numeral. Place the strawberries in a plastic bag. The student removes the strawberries from the bag and places the correct number on the vines. Make 10 vines, either in separate boxes or file folders or several in one box. This activity provides practice in counting objects one to one.

CONCRETE AND ABSTRACT MANIPULATIVES: AN APPROACH TO PROBLEM SOLVING

by Ruth Hadley

Problem solving in the primary grades can be approached through sorting, matching, ordering, sequencing, and classifying manipulatives before using paper and pencil. This can provide a learning experience that is both valuable and pleasurable for the students. The author is a sixth grade teacher, Los Padres School, Lompoc Unified School District, Lompoc, California.

Call them audiovisual materials, or games, or puzzles, or independent study materials for learning centers. Whatever they are, manipulatives provide for students in the schools, especially elementary students, a welcome change of pace. If the materials are mathematically sound, they provide valuable practice for the learning of math concepts. There is an effect, call it novelty, that appeals to most students. It could be that the students are not sophisticated enough to realize that they must use their math knowledge to work with the manipulative materials, so they think it's fun and stressfree.

Younger students explore the world in which they learn to recognize objects and to classify. The first such experiences are of a sorting type. A teacher can provide a bag of three colors of marbles and three boxes, and have the students sort. The same sort of task can be done with blocks of different colors, blocks of different sizes, or plastic animals of different species. Blocks of four different colors and two different sizes can be sorted into four boxes or just two; have students sort the blocks into the four boxes, and then take away two boxes and ask them to sort the blocks again. Students can also sort polygons according to the number of sides they have. If different colors are also introduced, the sorting can be done in different ways.

A variation from the task above is to have students sort out materials that do not belong to a group such as the examples in Figure 1.

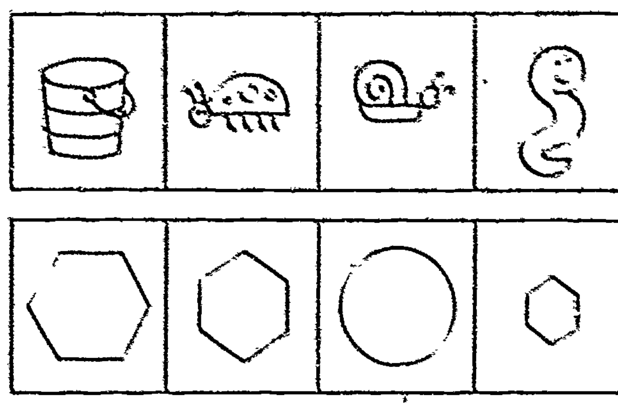


Figure 1

Students have to be involved mentally with the task of matching. One simple exercise is to cut squares of wallpaper from a sample book, mix them up, and have the student match the pairs. Another task for lower primary students is to provide them with many geometric figures such as triangles, squares, circles, rectangles, rhombuses, trapezoids, and such, each figure represented by either different colors or different sizes, and then have the students match them by one characteristic.

There is a board you can use with geometric figures that requires that you match figures horizontally by one difference, and vertically by two differences (see Figure 2). To make the task easier, require just one difference each way; to make it harder, require two or more differences each way.

Students should be able to order, or be given many opportunities to do so if they have difficulties. Ordering can be done by size, by weight, by age, by speed, or by degree. Teachers can make cards with pictures so that the students can put the cards in the proper order. Figure 3 offers a few examples of the types of activities students can do.

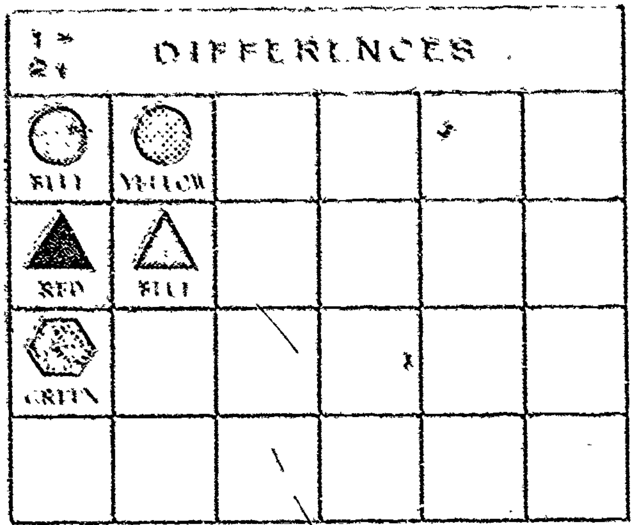
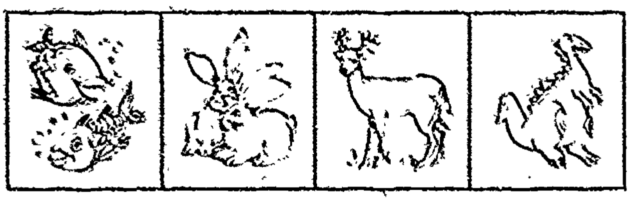
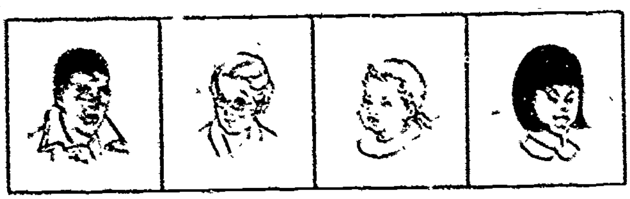


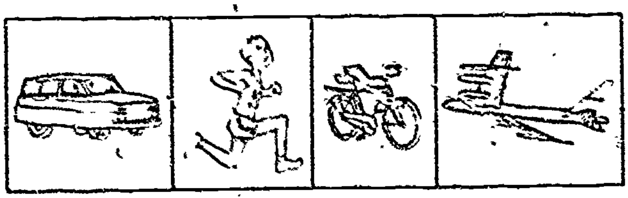
Figure 2



The one above has been ordered from smallest to largest. You do the others.



Youngest to oldest



Slowest to fastest

6	9	0	5	8	4

Numerical order

Figure 3

Sequence is another math concept that can be strengthened by practice with many activities. With younger students, physical exercises help to introduce them to sequence. The teacher starts a sequence like clapping the hands and snapping the fingers alternately. The next step would be to snap, clap, clap and ___ . Right, snap! To make it more involved: snap, clap, clap, slap knees, and repeat. The point is to start a pattern and let the students respond. If they are right, smile or praise them, and continue the pattern for a bit. More praise and then another pattern. Kindergarteners and first graders love this sort of activity as a change of pace, but they are making decisions while they are doing it. You can make the patterns more elaborate by having the students stand, hop, make a verbal response, and so on.

An individual approach to sequence solving can be accomplished by making up cards with a pattern on them with the last space vacant for the student to complete the pattern (see Figure 4). Plastic toys, wooden beads, beans, and all manner of things can be used. Extra materials to use to complete the sequences should be available to the students in a container.

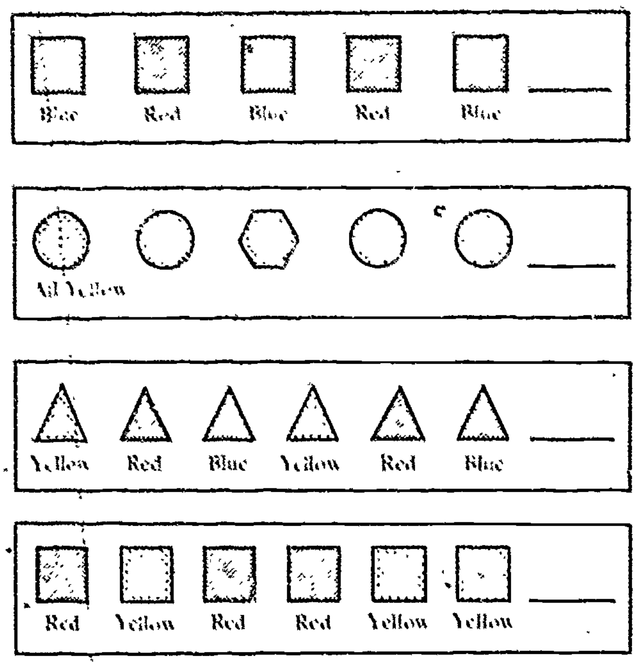


Figure 4

To the casual observer, all the activities discussed so far may seem to have very little or no relation to math, but as Piaget has said in his many books and articles, students must relate to what they are doing; they learn to understand by seeing, by playing with real objects, and by manipulating them. The understanding comes from within the student. After many activities such as those discussed in this article, which the student is able to respond to in a positive manner, number sequencing can be introduced. This can be done with practice sheets such as that shown in Figure 5.

1, 0, 1, 0, ___, ___, ___

1, 2, 3, 4, 5, ___, ___, ___

18, 17, 16, 15, ___, ___, ___, ___

3, 5, 7, 9, ___, ___, ___, ___

3, 6, 9, ___, ___, ___

2, 3, 5, 8, 12, ___, ___, ___

2, 4, 8, 16, ___, ___

1, 8, 27, 64, 125, ___

1/2, 1/4, 1/8, 1/16, ___

1/5, 1/10, 1/20, 1/40, ___

___, 16, ___, 32, 40, ___, 56, ___

___, ___, ___, ___, ___, 60, 70, ___

40, 20, 24, 12, 16, 8, ___

1, 10, 100, ___, 10,000, ___, ___

1/4, 1/20, 1/100, ___, ___

3/6, 1/2, 3/12, 1/4, 8/12, ___

1/4, 2/8, 3/12, 4/16, ___, ___

Figure 5

Sorting, matching, ordering, and sequencing have been treated here as ways of helping the student make a decision concerning certain materials. Each of these activities infers that a student has been forced to make a decision which is the process necessary for problem solving. The last of these topics is classifying.

Piaget suggests Venn diagrams for students prior to working with the basic operations to provide the foundation necessary for the understanding of the basic operations. Union, intersection, and subset, as well as the idea of negation, can be graphically shown to the child.

The disjunction shows the "and" of logic; the conjunction shows the "or" (Figure 6). The idea of a negation seems to be understood more easily by students if it is introduced with concrete objects. Disjoint sets illustrate the absence of sharing any elements; subsets illustrate Piaget's inclusion relation (Figure 7). Attribute blocks or teacher-made materials can be used in these exercises. A group of cards can be made, each listing an attribute. Students then select cards at random to put in the circles. As they put the objects in the circles, they may discover overlappings of attributes.

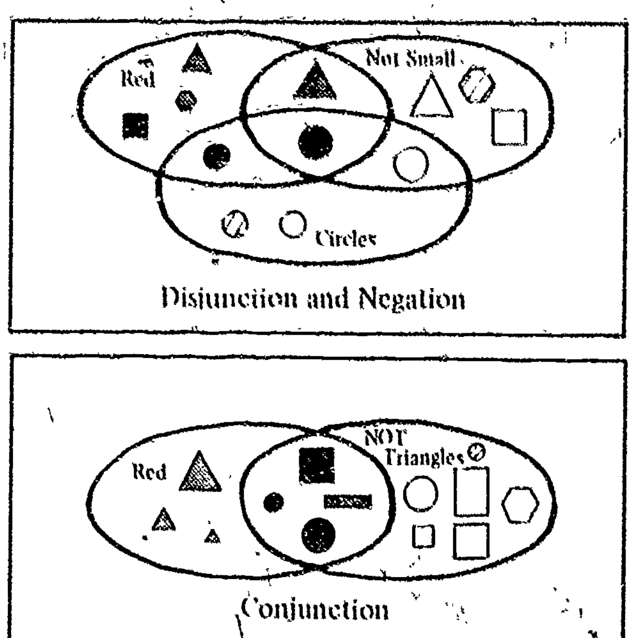


Figure 6

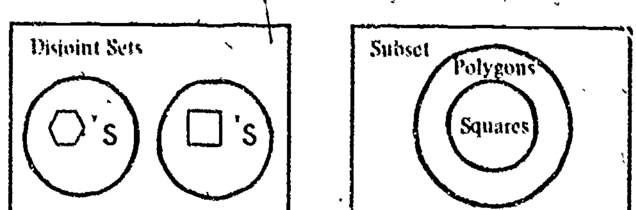


Figure 7

In essence, each of the foregoing exercises requires students to make decisions, not recall number facts. Perhaps this type of activity should be encouraged and hopefully will help students with the problem-solving strand of math.

Piaget has stated that until approximately seven years of age, students are at a prelogical and prenumerical level of understanding. This would seem to preclude the teaching of math and problem-solving using the basic operations at any level of abstraction. Teachers are urged to provide manipulatives of commonplace objects so that students have many opportunities to manipulate them. This activity seems to combine visual and tactile senses to provide understanding. Much that is done in math in the classroom today is introduced too early for the students, and the students, if they are able to cope at all, do so in a rote memory way.

After all, whenever adults have difficulty explaining concepts, they usually say, "Let me *show* you," or, "I'll *draw* you a diagram." We don't think it unusual for young students to play with blocks and other manipulatives to begin the study of numbers. So why shouldn't all of our students, regardless of their ages, start at the concrete or the manipulative level as they attempt to learn new math concepts?

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THE ARITHME-LADDER

by Robert L. McGinty and Theodore A. Eisenberg

The ARITHME-LADDER is a manipulative device that can be used with primary students to aid in the instruction of counting and the four basic operations. It can also double as a balance. The authors are affiliated with Northern Michigan University, Marquette. A shortened form of their article was published in The Instructor, January 1975.

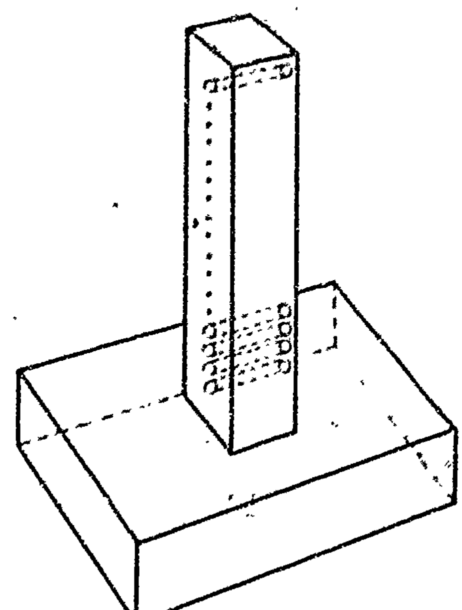
The use of manipulative aids in the teaching of arithmetic is by no means a new or even recent idea. Pestalozzi (1746-1827) used beans and pebbles in teaching children to count. Wentworth (1835-1906) used sticks in bundles of 10 to illustrate the idea of place value. In fact, the use of manipulative aids in mathematics instruction can be traced back at least to the time of ancient Greece (Lanczos, 1970). But the great number of teachers using manipulative aids is a recent trend, and its impact is easily seen by visiting almost any elementary school classroom.

There are many reasons for advocating the use of manipulative materials in teaching arithmetic. Davis (1966) states

> The rationale in support of physical materials and experiences comes from three sides: the cognitive psychology of Jean Piaget, the reality-authoritarian dualism of modern science and modern psychoanalytic theory, and the nature of mathematics in today's world.

Trying to implement Piaget's theory of learning in classroom settings has been a major rationale for the use of physical materials. Piaget's theory of learning asserts that children go through several stages of mental development, one of which is the concrete operational stage. Davis (1966), Dienes (1960), Pottenger and Leth (1969), Kallet (1967), and others have observed that effective progress through the concrete operational stage is enhanced by structuring situations in which the students can play with and manipulate physical objects.

One manipulative device of unknown origin is the *ARITHME-LADDER*. It has been used successfully in grades K to 3 as an aid in counting, adding, finding the missing addend, subtracting, multiplying, and dividing and also as a balance. The *ARITHME-LADDER*, as shown in Figure 1, can be made with only two pieces of wood.



Specifics (Suggested Dimensions)

Base: 8" X 8" X 1"

Ladder: 23" X 2" X 1" with 21 holes (¼ diameter) drilled 1 inch apart

Pegs: 3" wooden dowels are used for pegs

Figure 1

Also, the numerals 1 through 21 can be written on the side of the ladder, if so desired. Types of *ARITHME LADDER* activities will now be discussed.

COUNTING

1. Have the student put the pegs in the holes one at a time, beginning with the first hole, while counting out loud.
2. Start with all the pegs in the holes and push the pegs to one side, one at a time, while counting out loud. *See Figure 2.*

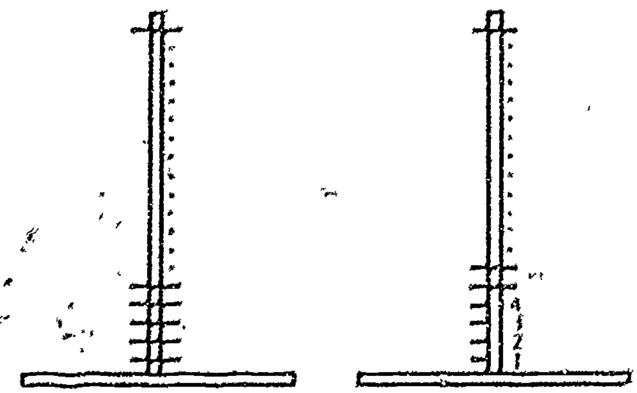


Figure 2

3. To count by 2's, 3's, etc., the students can proceed as above, but they can push 2, 3, etc., pegs to the side each time.
4. In a manner similar to the above, the students can count backward from a predetermined number (less than or equal to 21).
5. To give practice in writing numerals, the students could write the numeral as the pegs are pushed to one side.

ADDITION

1. An addition problem such as 2 + 3 could be done by placing 2 pegs in the holes (beginning with the first hole) and then placing 3 more pegs in the holes to arrive at the answer of 5.
2. A variation of adding 2 and 3 can be accomplished by starting with all the pegs in the holes and then pushing 2 pegs to one side and pushing 3 pegs to the other side to arrive at the answer of 5. See Figure 3.

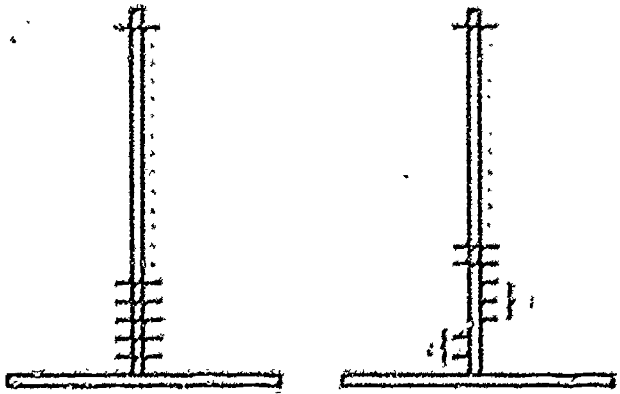


Figure 3

3. Missing addends, such as 2 + [ ] = 5 can also be found with the *ARITHME LADDER*. One way is to have the student begin by placing 5 pegs in the holes (beginning with the first hole) and then push 2 pegs to one side, noting that the 3 remaining pegs constitute the answer 3. A second method is to start with all of the pegs in the holes and then, beginning at 5, push 2 pegs in one direction, noting that the 3 lower pegs constitute the answer.

SUBTRACTION

1. A subtraction problem such as 5 - 3 could be done by placing 5 pegs in the holes (beginning with the first hole) and then, starting with the fifth peg, remove 3 pegs to arrive at the answer of 2.
2. As in addition, a variation of the subtraction problem 5 - 3 can be done by starting with either all the pegs in the holes or just the first 5 pegs in the holes. Then, beginning with the fifth peg, the student could push 3 pegs in one direction, noting that the lower 2 pegs constitute the answer of 2.

MULTIPLICATION

1. A multiplication problem such as 2 X 3 can be done as a repeated addition problem. Beginning with the first hole, have the student place 2 groups of 3 pegs (or 3 groups of 2 pegs) in the holes, one group at a time.
2. A variation of a multiplication problem such as 3 X 2 can be done by starting with all the pegs in the holes. Then, beginning at the first hole, alternately push a group of 2 pegs to the left, then

a group of 3 pegs to the right [illegible] until the required number of groups has been reached.

DIVISION

1. A division problem such as 12 ÷ 3 can be done as a repeated subtraction problem. Have the student put pegs in the first 12 holes. Then, starting at the twelfth hole, have the student take out a group of 3 pegs and place them on his or her desk, then another group of 3 pegs, etc., until he or she has removed all the pegs. The number of groups of 3 on the desk is the answer to the problem.
2. As an alternative method to find the answer to 12 ÷ 3, have the student again place pegs in the first 12 holes (or every of the holes). Starting with the twelfth hole, have the student push groups of 3 pegs alternately to the left and to the right. The answer is the number of groups of 3 showing on the *ARITHME-LADDER*.

BALANCE

You may have been wondering why 21 holes are drilled in the ladder. To convert the ladder to a balance, the extra hole is needed so that the balance will have 10 pegs on each side. To make the ladder into a balance, another piece of wood must be added as a stem. See Figure 4. Typical balance activities can be performed by using washers as the balance weights (Sealey).

While the *ARITHME-LADDER* is by no means a cure-all for the problems that students encounter in learning the basic operations, it can be a useful and inexpensive addition to a teacher's repertoire of instructional aids.

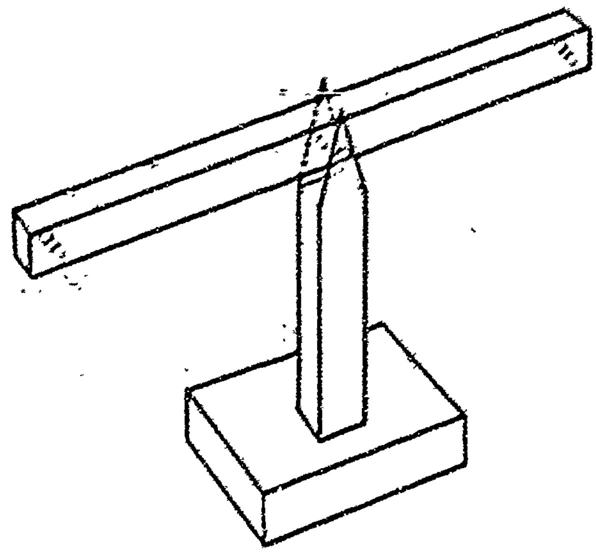


Figure 4

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CHIP TRADING AND PAPY'S COMPUTER

by John Firkins

John Firkins is in the Mathematics Department of Gonzaga University, Spokane, Washington. This material was originally published in Washington Mathematics in 1977. Both activities are useful in working with the concepts of place value and the decimal system with elementary students.

Rationale

Although both of these activities may have specific objectives, each is part of the elementary teacher's collection of manipulatives that help students move from the concrete to the abstract in a realistic and satisfying way. Each manipulative is but one of the ways in which understanding develops, and it is, therefore, important to provide many parallel experiences in order for the student to see that the ideas and concepts are really and truly independent of the given manipulative.

To demonstrate understanding of a concept or operation of arithmetic, a student needs to (1) write the basic fact associated with the given manipulative and (2) interpret the basic fact by modeling that number sentence with the physical model as provided by the given manipulative. Such a mathematical device is not a toy. It is something to be used by a skilled teacher to develop a concept, and it becomes the means of evaluation by which the teacher tests to see if understanding has been achieved.

CHIP TRADING

Objectives Chip trading is used at any grade level to develop the concept of place value or to reinforce it after students have been exposed to other activity-oriented experiences such as bean sticks and/or multibase blocks. As students actually make "fair trades" with colored chips, they gain an understanding of one-to-one correspondence, many-to-one correspondence, place value, regrouping, and some of the basic number operations. Thus, chip trading is another means to create concrete experiences for students that lead to abstract formulation later on.

Materials Colored chips—e.g. plastic counters, colored posterboard—in at least four colors, dice, and paper tills. Each student will need a paper till, a regular sheet of paper ruled and labeled as in Figure 1 which shows the pattern.

RED	BLUE	YELLOW	WHITE

Figure 1

Any other heading may be used, depending on the colors of the chips you have on hand. The following activities are only a portion of those possible using the above material.

Banker Games

1. Every group of four or five students elects a banker from among themselves. The banker then places the colored chips in the appropriate divisions on his or her till. Each player takes a paper till and a die. The game begins with the banker deciding the value assigned to each chip. For instance, the value may follow the exchange listed below.

3 white	=	1 yellow
3 yellow	=	1 blue
3 blue	=	1 red

The player on the banker's left begins by rolling his or her die and asks the banker for the number of white [illegible] the trade [illegible]. The game continues, making all possible trades until the first to reach [illegible] is declared the winner.

2. The game differs from game 1 in that each student is given a red chip and must get rid of it by paying the banker the number of white chips shown on the die with each turn. The first player to rid himself or herself of the red chip through "fair trades" and to pay the banker until no chips are left is declared the winner. The player must go out exactly.
3. The game differs from game 1 only because every third round becomes a "pay" round. The game continues exactly as before except that the first and second turns are "collect" from the banker and the third is "pay" the banker the amount shown on the die. The fourth and fifth turns are "collect" and the sixth a "pay," etc.
4. The banker gives each player one chip of each color. Each throw of the die tells how many white chips the player must pay the banker. The banker makes change and sees that the correct number of chips is paid. The first player to get rid of all his or her chips is declared the winner. The player must go out exactly.

Banker games should be continued until the students are secure with counting and making trades. Remember that trades may vary in each of these games. If a 10-1 trade or other large numbers are used, then more than one die should be used.

PAPY'S COMPUTER

Mme. Frederique Papy is a mathematics educator at the Centre for the Pedagogy of Mathematics in Brussels. The method she uses to introduce students to mechanical and mental arithmetic employs the distinct advantages of the binary system over other positional systems, while at the same time keeping the student immersed in the decimal system.

Papy's minicomputer is a two-dimensional abacus. It uses the binary system on cards that are arranged according to the decimal system. Cards can be glued on a file folder as shown in Figure 2.

The addition of whole numbers is accomplished automatically by "playing" the machine. The same is true for doubling, which is a primitive experience for children and is fundamental to Papy's computer. Memorizations, which so often form a distasteful part of learning to compute, are held at a minimum. Addition of small numbers is accomplished by simple rules, a purely binary system when the

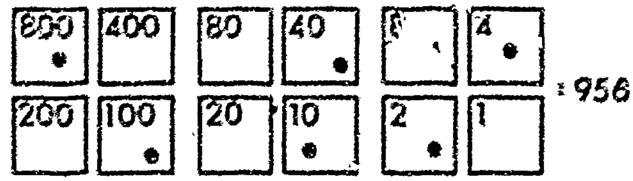


Figure 2

sum is less than 9 and a mixed decimal-binary system in other cases. Thus, from the beginning the students are initiated into a positional system of numeration.

The version of Papy's computer presented here has one rule: no card can have more than one counter on it. Therefore, changes must be made to accommodate this situation as it occurs during the operation of addition or subtraction.

Papy's computer is easily introduced as an independent learning activity. Cards written for the computer and its operation can be introduced as follows.

Independent Exploration I

Title Card

Title	Playing Papy's computer
Objective	When you complete these cards, you will be able to "play" the computer.
Materials	Papy's computer and some counters
Hints	None

Card 1

1. Place [illegible] counters on the 1 card.
2. Pick up two counters, discard one, and place the other on the 2 card.
3. Continue step 2 until you can no longer pick up two counters.
4. Pick up two counters from those on the 2 card, discard one, and place the other on the 4 card.
5. Repeat step 4.
6. Pick up the two counters on the 4 card, discard one, and place the other on the 8 card.
7. Pick up the chip on the 8 card and one chip from the 2 card, discard one, and place the remaining chip on the 10 card.
8. Repeat steps 4, 5, 6, and 7 until you are no longer able to place a counter on the 8 card.
9. Pick up two counters on the 10 card, discard one, and place the other on the 20 card.

10 You have now "played" [illegible] the least number of [illegible]

6 Pick up your [illegible]
7 See the teacher for the [illegible]

Independent Exploration II

Title Card

Title	Adding whole numbers using Papy's computer
Objective	When you complete these cards you will be able to add to sums of 999
Materials	Papy's computer, counters, pencil, paper
Hint	None

Card 1 Set your computer before you.

1 Enter 6 on your computer
2 Enter 7 on your computer
3 Play the machine
4 On your paper draw a picture of the position of the counters after playing the machine
5 Write the numeral representing the picture you have drawn
6 Clear the counters from the card

Card 2

1 Enter 24 on your computer
2 Enter 49 on your computer
3 Play the machine
4 On your paper draw a picture of the result of playing the machine
5 Write the numeral representing the result on your paper

[illegible]

1 Enter 465 on your computer
2 Enter 347 on your computer
3 Play the machine
4 Draw a picture of the result on your paper
5 Write the numeral representing the result on your paper
6 Pick up your counters

Card 4

1 Enter 37 on your computer
2 Enter 123 on your computer
3 Enter 19 on your computer
4 Enter 41 on your computer
5 Enter 279 on your computer
6 Enter 53 on your computer
7 Play the machine
8 Draw a picture of the result on your paper
9 Write the numeral representing the result on your paper
10 Pick up your counters

Card 5

1 Write down in a column four numerals less than 200
2 Enter each one on the computer
3 Play the machine
4 Record the result
5 See the teacher for comments

SEQUENCE STRIPS

by Betty J. Sternberg

The author is Assistant Director of Rescue, a regional educational service center in Bridgewater, Connecticut. Her activities can be used with students in the primary grades and up, depending on the complexity of the task cards selected.

There is a multitude of inexpensive, easily available materials that one can use to put together activities that can help the student exercise and extend her or his ability to reason inductively. One such material is colored construction paper cut into strips of five distinct sizes. By manipulating the set of construction paper strips on task cards, the student can discover logical and spatial relationships essential to mathematical understanding.

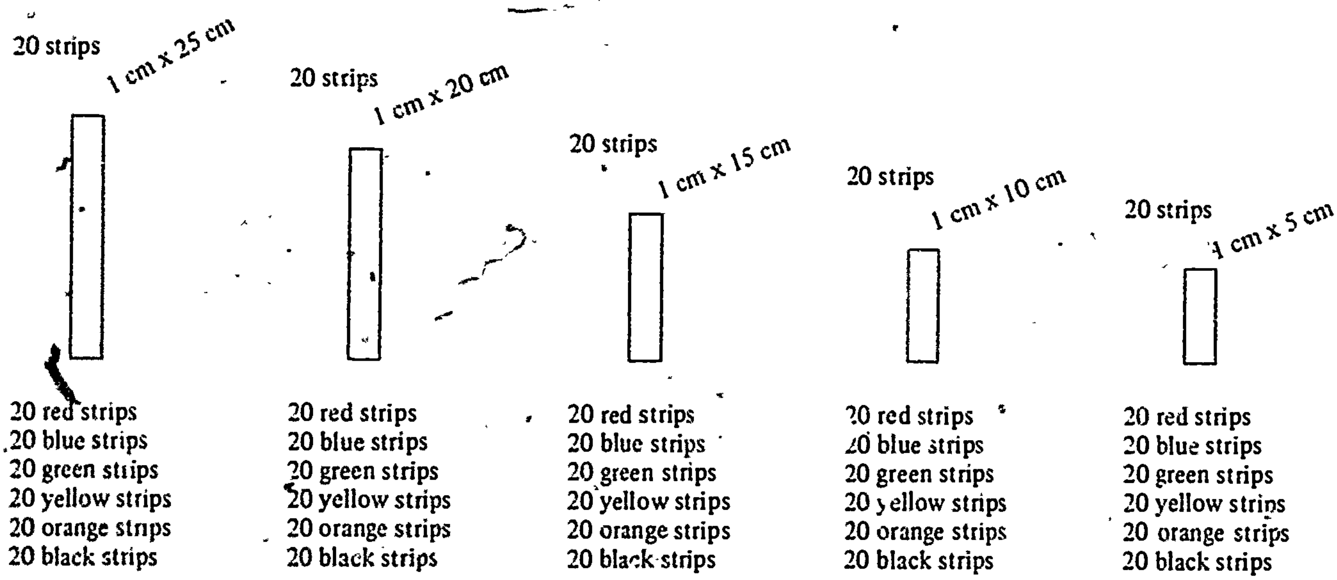
THE STRIPS

Six specific colors should be chosen. Each piece of colored paper must be cut into strips of five distinct sizes, as follows: 1 cm X 5 cm, 1 cm X 10 cm, 1 cm X 15 cm, 1 cm X 20 cm, 1 cm X 25 cm. Cut at least 20 strips of each size in each color. Your set will look like that shown in Figure 1.

THE TASK CARDS

Task cards are easily made with crayons, markers, or pencils colored to match your construction paper colors.

The first tasks you create should present the beginnings of relatively simple patterns for which the student must ask if one strip differs from the next in color or size, or both color and size. For example, your first card might



TOTAL of 100 strips of each color listed above.

Figure 1

look like that in Figure 2. The student must look at the pattern and extend it with his or her own set of colored strips.

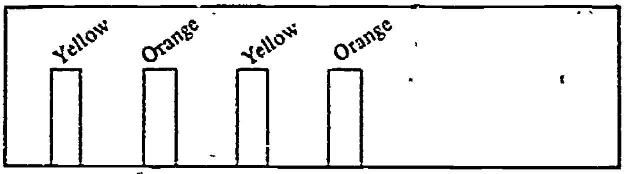


Figure 2

Your first patterns should all be presented in a left-to-right sequence with the student continuing it to the right. It is sometimes useful to direct the student to say the color sequence out loud in order to help discover the pattern. Always be prepared for an answer you did not expect! For example, a student once completed the card above as follows:

Yellow, Orange, Yellow, Orange, Red, Green, Red, Green

When asked what she/he had done, the student indicated, in his/her own language, that the first four strips alternated in color; thus, she/he picked four other strips that alternated in color. When asked to continue the pattern further, she/he did the following:

Y, O, Y, O, R, G, R, G, Y, O, Y, O, R, G, R, G

The student had, indeed, discovered the given pattern, and rather than merely continuing it, she/he extended it.

As can be seen, a primary characteristic of the task cards you will make is that there is no one logically necessary answer. If the student can explain intelligently what she/he has done or, more particularly, what rule she/he discovered and attempted to follow or extend, then the student has found his/her own "good" solution. The solution may differ from yours or that of another student and still be valid.

Cards can be made more difficult by altering both size *and* color at the same time. Examples are shown in Figure 3.

Cards can be made even more difficult by varying size, color, *and position* as shown in Figure 4.

Another way to alter the cards is to place the pattern in the center of the card and ask the students to build out to the left and to the right of the pattern. While reinforcing the left-to-right sequence is important for reading, reinforcing the right-to-left sequence is important for math. So, a card might look like that in Figure 5. It will be useful to have the student say the pattern out loud as she/he tries to extend it from right to left.

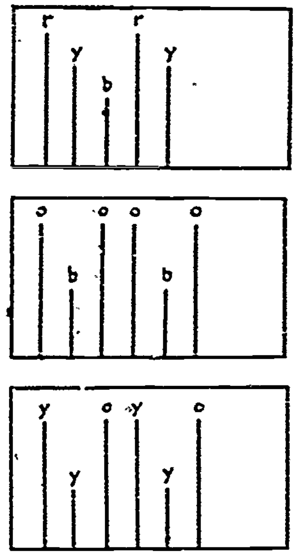


Figure 3

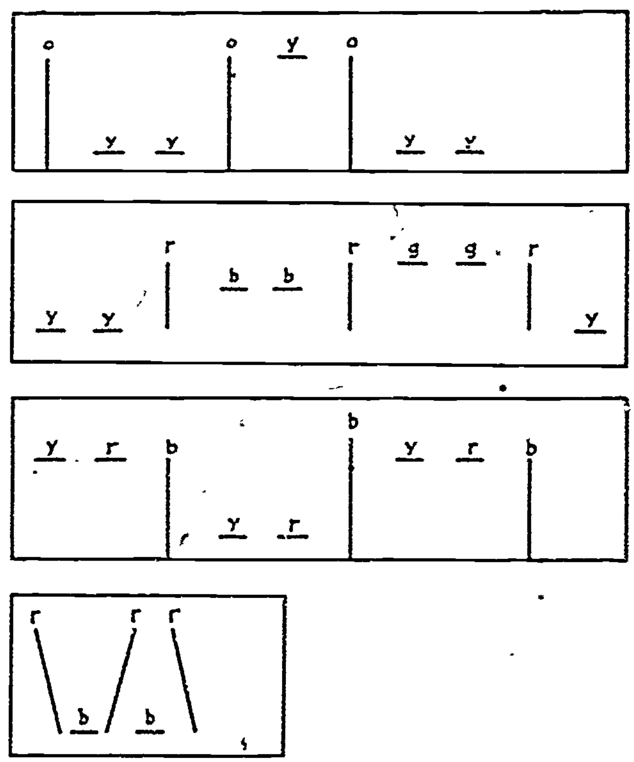


Figure 4

In order to extend the logical process more fully, it is useful to present matrix patterns on the cards. In order to

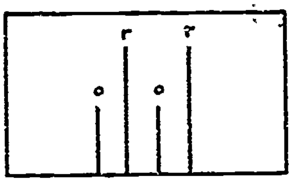


Figure 5

complete a matrix, the student must first discern both the horizontal patterns in each row *and* the vertical patterns in each column. She/he must then draw an inference as to which strips are missing. Once again, it must be stressed that there is no one logically necessary answer to each card

An example is shown in Figure 6.

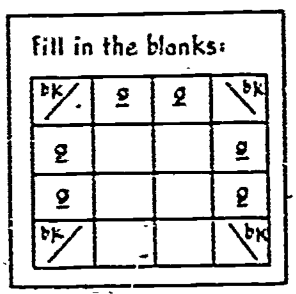


Figure 6

Two acceptable answers are shown in Figure 7.

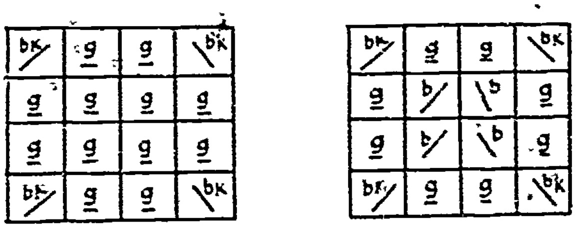


Figure 7

One type of task card that is particularly stimulating presents rotational mappings. A rotational mapping card might look like that in Figure 8.

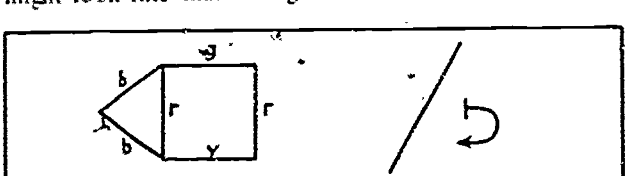


Figure 8

In order to complete a rotational mapping card, the student must be able to retain the spatial and color configuration of the strips, while moving them into upside down and sideways positions.

The symbols ↻ and ↺ can be introduced to indicate 180° (upside down) rotations. First, the student should be instructed to envision what the given picture would look like if it were upside down. Then, she/he should build the upside down version with the strips. The student can "check" his/her answer by placing another set of strips over the original picture, turning it upside down, and comparing it to his/her answer.

The symbols ↴ and ↰ can be introduced to indicate 90° (sideways) rotations. The student should place the strips over the original picture and give them a quarter-turn in order to complete the cards. Two samples are given in Figure 9.

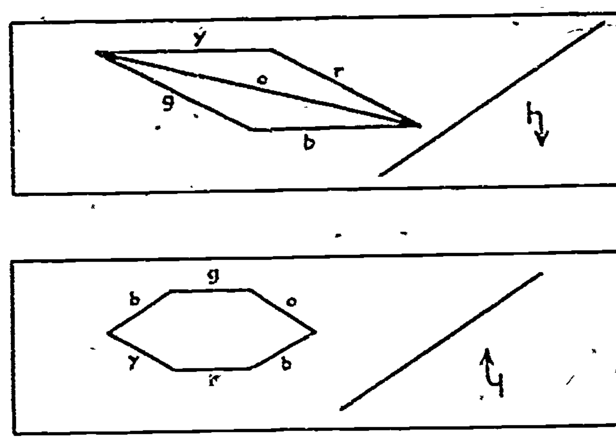


Figure 9

Finally, cards that deal with mirror symmetry are exciting and can be used either with or without mirrors as aids In order to complete a card, the student must envision what the mirror image of the given strips would look like if a mirror were placed along the black line drawn on the card. She/he could then actually use a mirror to "check" the answer. If a student has had little or no experience with mirrors, this activity should *begin* by allowing the student to view the mirror image of the given drawing in a mirror The student should look into the mirror, note the color and placement of strips in the mirror, and then attempt to build that image on the other side of the black line. Cards would look like those in Figure 10.

A FINAL NOTE

The importance of providing the student with manipulative materials and tasks that help exercise and extend

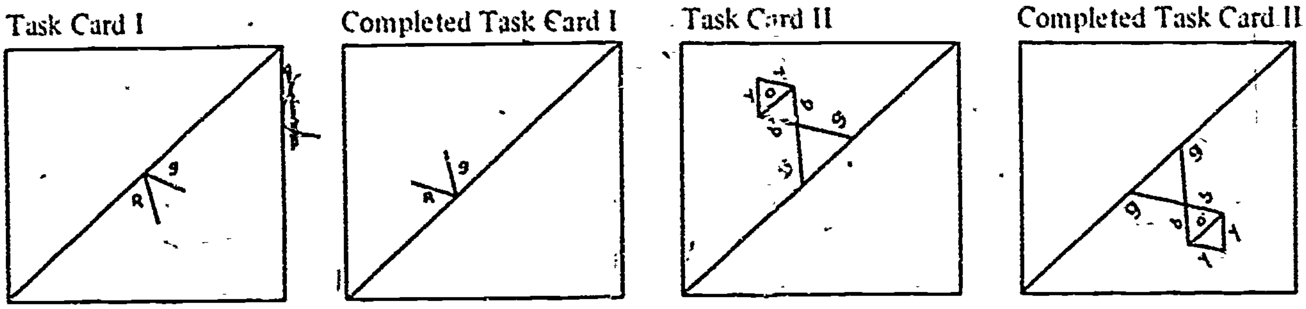


Figure 10

his/her ability to think logically cannot be overemphasized. Colored construction paper strips, used in conjunction with task cards, offer an enjoyable, educational experience. These simple materials can provide one of the many stepping stones by which the student, through sight and touch, can begin to make sense out of the environment.

THE LONG DIVISION BOARD

by Rebecca S. Nelson

The long division board is designed to be used in conjunction with any place value material. Through its use, the long division algorithm can be meaningfully taught to intermediate level students. The author is an Associate Professor of Mathematical Sciences at Ball State University, Muncie, Indiana.

Many mathematics educators have noted, and all fourth or fifth grade teachers know for sure, that long division of whole numbers is a difficult process to teach. As teachers, we want to make the algorithm meaningful, but to do so often means that we must abandon the more traditional algorithm and use some form of the subtractive algorithm (Heddens, 1975; Swart, 1975). Although the subtractive algorithm is fine, and in many cases the most appropriate, the traditional algorithm can also be taught in a meaningful way with the help of concrete materials and the long division board.

The division board can be used with any material that embodies the place value concept: fake money in decimal units (one-, ten-, hundred-, and thousand-dollar bills); bean sticks or base 10 blocks; color-coded chips (Davidson, Galton, and Fair): color-coded place value pocket chart; painted rocks or other variations.

In the beginning, work with the long division board, a color-coded device, and appropriate color-coded numeral cards is most helpful.

MAKING THE BOARD

To make the long division board, choose a piece of scrap lumber or masonite from your lumberyard about 50 inches long and 24 inches wide. You will also need a small can of white enamel paint, 33 cup hooks, and some black electrician's tape. Figure 1 shows a sketch of the board itself. Place the cup hooks about ½ inch farther apart than the length and width of your numeral cards. You will also need numeral cards from 0 through 9 in each color of your color-coded material and numeral cards in white for the divisor. Figure 2 shows a sample numeral card.

THE DIVISION BOARD

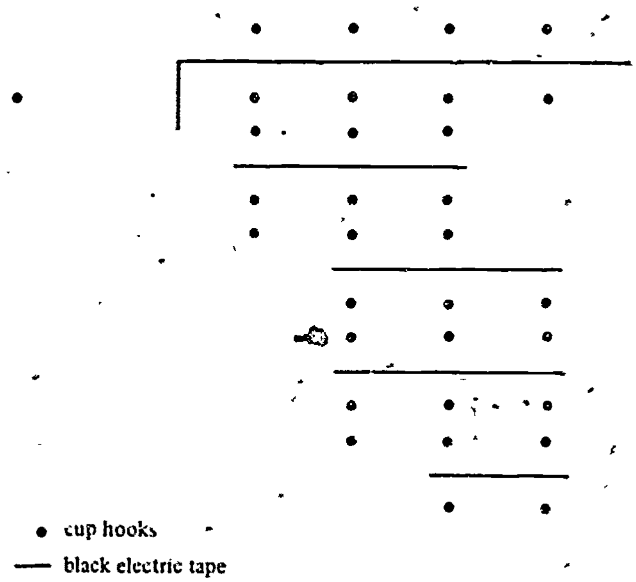


Figure 1

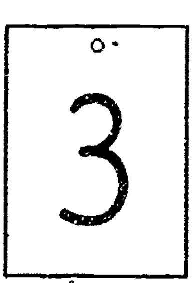


Figure 2

USING THE BOARD

The division process here is similar to that reported by Margaret W. Maxfield (1974); however, here each student or pair of students should already have enough concrete materials to solve the example at their seats. The role of recording on the division board may be handled by the teacher at first, but later it may be passed to a student. For purposes of illustration, the chip trading material and measurement division (making sets of ◯) will be used.

Suppose that you want to solve 537 divided by 4 using the long division board. Ask the students to set up 537 on their chip tills (see Figure 3).

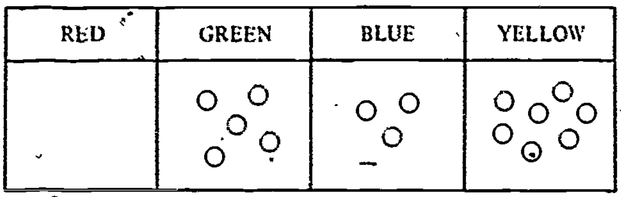


Figure 3

Record the example on the division board by hanging the appropriate color-coded numeral cards on the cup hooks (Figure 4).

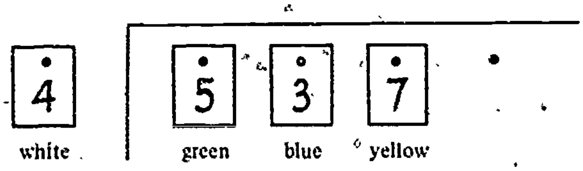


Figure 4

Now ask: "What does *divided by 4* mean? How many sets of 4 can you make in green chips? Make all the sets you can. How many green chips are left?"

The students should do this on the chip till (Figure 5).

Record this on the division board (Figure 6)

Ask. "What can we do with the green chip left? How many blue chips are worth 1 green chip? Show this with your chips. How many blue chips did we have before the trade? How many will we have altogether after the trade?" (See Figure 7.)

Record this on the division board by covering the 1 in green with a 1 (ten) in blue and showing the 3 in blue before the trade, making 13 in blue (Figure 8).

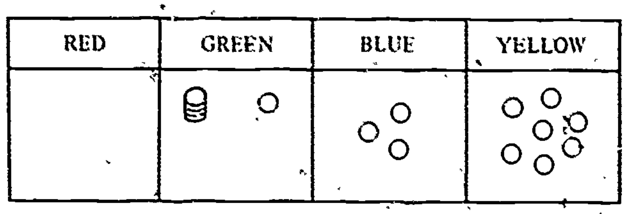


Figure 5

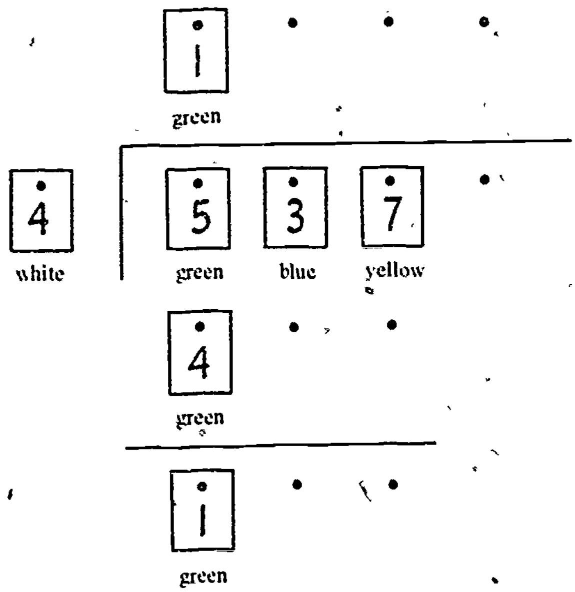


Figure 6

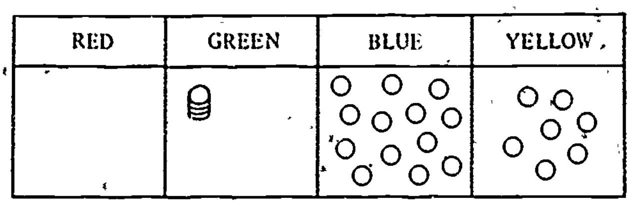


Figure 7

Ask. "How many sets of 4 blue chips can we make? Show this on your chip till." (See Figure 9.)

Record this on the division board by placing a blue 3 numeral card above the blue column on the division board (Figure 10).

Ask. "How many blue chips did this use? How many blue chips are left? Show this on your chip till." (See Figure 11.)

Record this on the division board (Figure 12).

Ask. "What should you do with the 1 blue chip left? How many yellow chips is 1 blue chip worth? How many

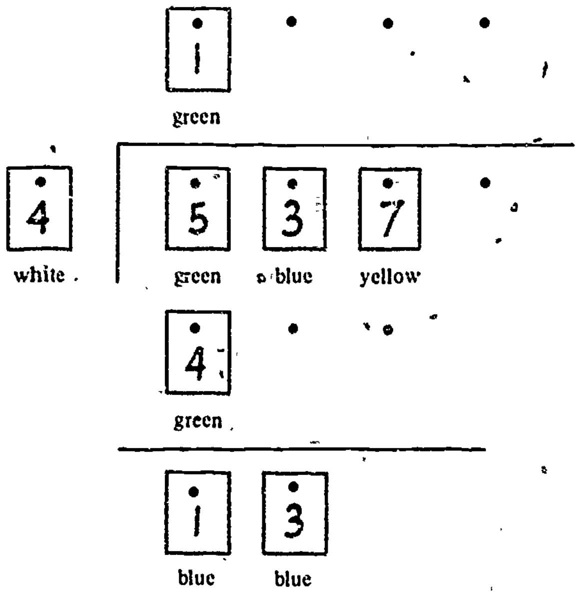


Figure 8

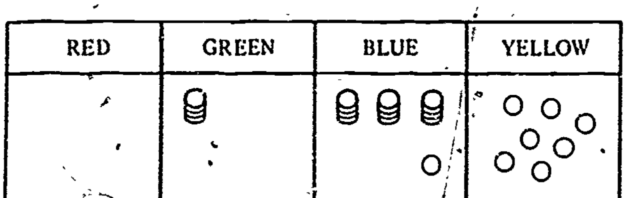


Figure 9

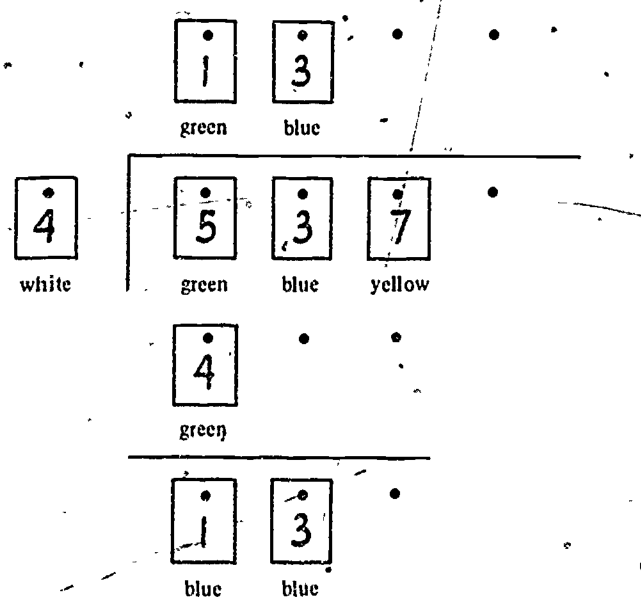


Figure 10

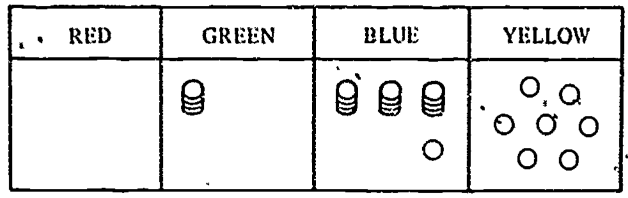


Figure 11

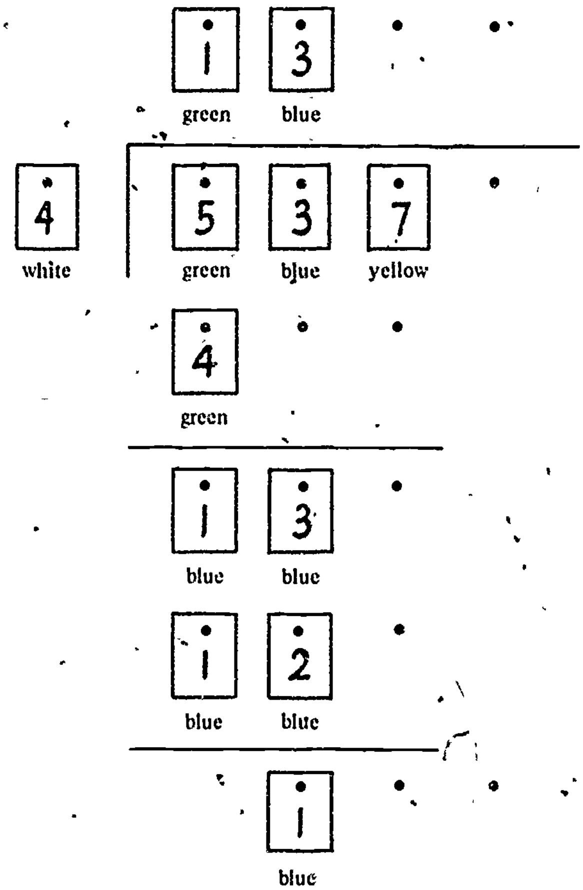


Figure 12

yellow chips, did we have before the trade? How many will you have total after the trade? Show this on your chip till." (See Figure 13.)

The teacher should record this on the division board by covering the 1 in blue with a 1 (10) in yellow and showing a total of 17 in yellow (Figure 14).

Ask: "How many sets of 4 yellow chips can you make?"

Record this on the division board (Figure 15).

Ask: "How many of the yellow were used in making the 4 sets? How many yellow chips are left?" Have the

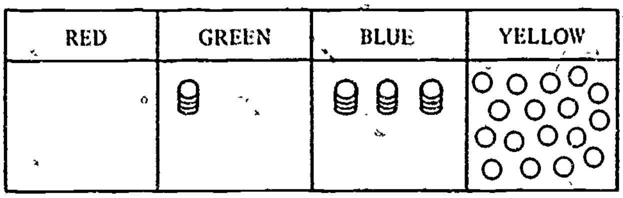


Figure 13

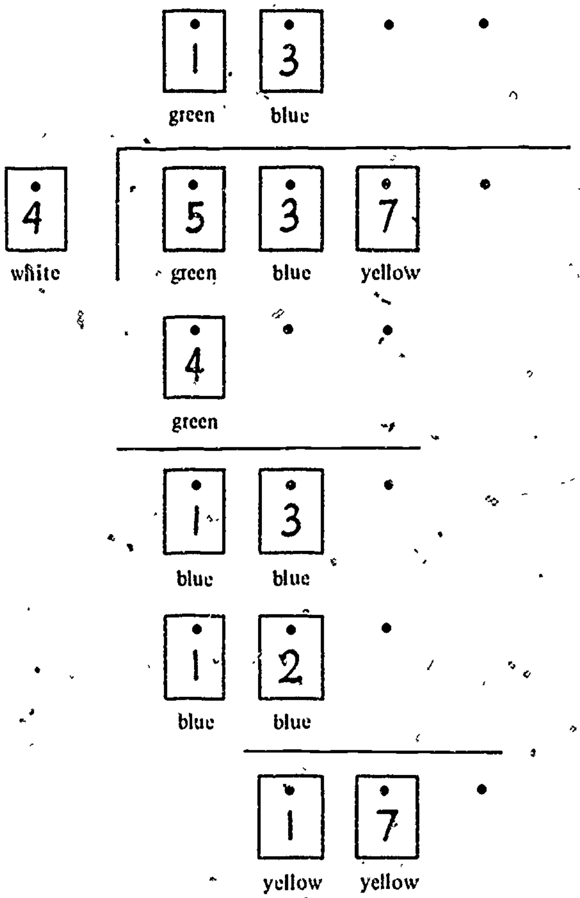


Figure 14

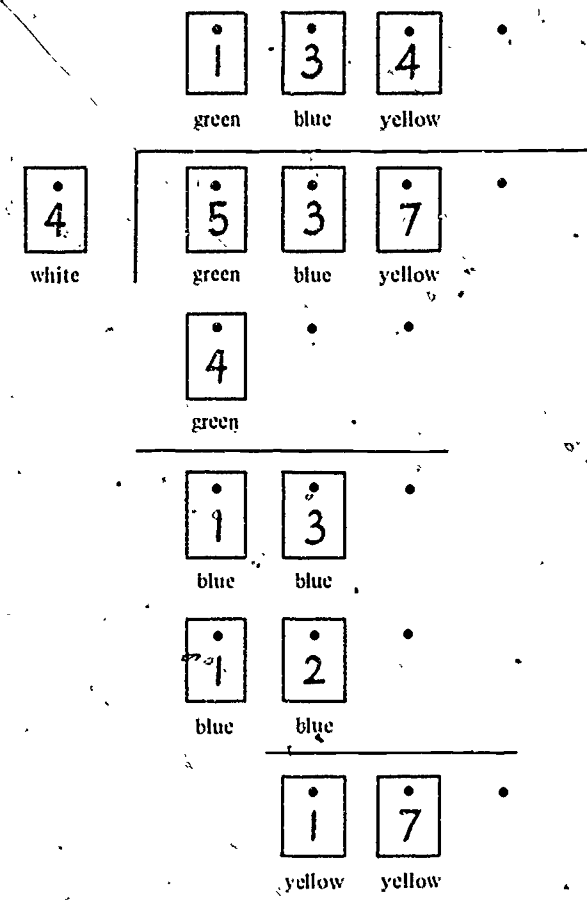


Figure 15

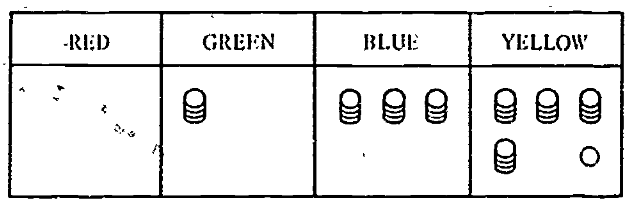


Figure 16

children show this on their chip till (Figure 16).

Record this on the division board. Ask questions about each of the numbers shown on the division board (Figure 17) and what you did to get them.

In the developmental stages while the concrete materials are being used, the examples should be carefully chosen to avoid trading in more than 3 chips of any one color for 10 chips each of the next color to the right. If this is not done, the students get sidetracked in counting and lose sight of the division. When solving larger examples at a somewhat later stage, the wording of the concrete materials may still be used, for example, to divide 4582 by 21, the teacher could ask, "How many sets of 21 could we make in red? What would we do? How many greens will we have after trading? How many sets of 21 can we make with 45 greens? How many greens will that use? How many greens would be left? What should we do with the 3 greens left? How many blues will we have after trading?" These questions can still be used after the color coding on the division board is no longer necessary and the traditional algorithm is then recorded either on paper or on the chalkboard.

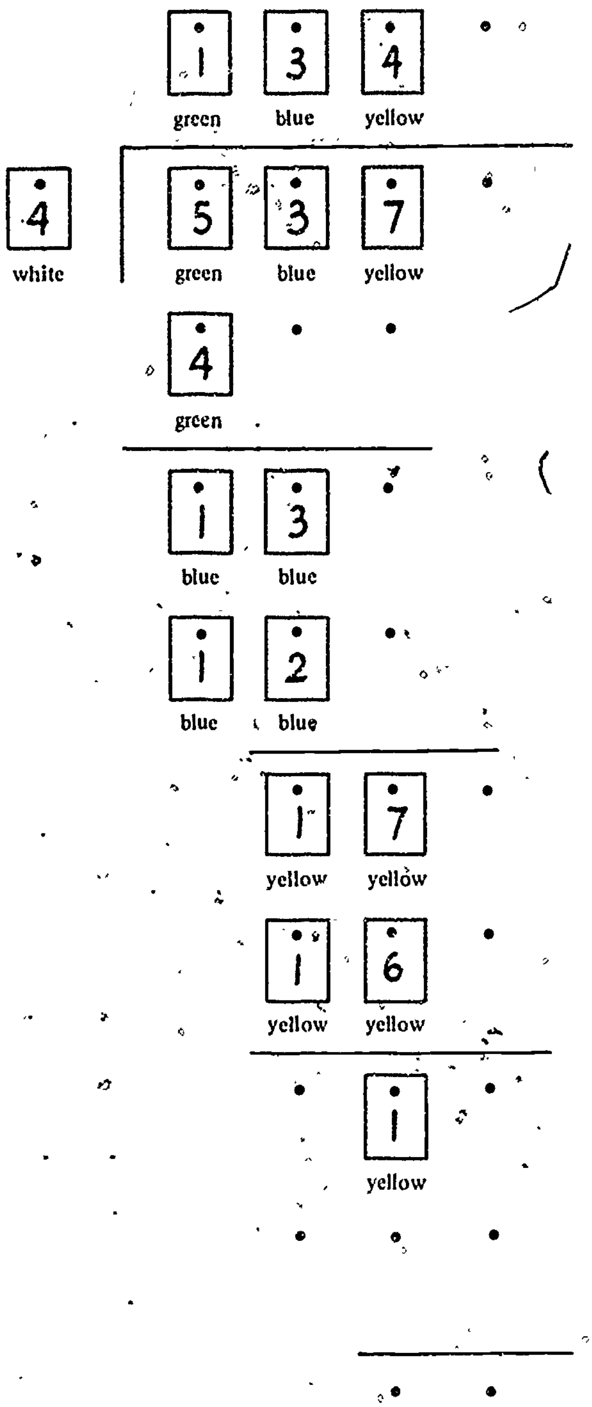


Figure 17

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AN ACTIVITY FOR EXPERIENCING AREA AND PERIMETER

by James H. Jordan

In 1975 the National Assessment of Educational Progress found that students and adults had a limited problem-solving ability, especially in the topics of area and perimeter. The author suggests the following activity as a means for elementary students to experience the concepts of area and perimeter, before the formulas are introduced. He is affiliated with Washington State University in Pullman as Professor of Mathematics; Chairman, Program in Science and Math Teaching; and Director, Mathematical Scholars Program.

Acknowledgement. As a consequence of attending several workshops conducted by Wayne Peterson, Basic Skills—Mathematics, Seattle Public Schools, the ideas were born that have culminated in this activity. For the inspiration—thank you, Wayne!

INTRODUCTION

The National Assessment of Educational Progress (NAEP) has reported that 73 percent of the adult public could not solve a simple area problem and that 73 percent of the adult public could not figure the area of a square, given the perimeter (Figure 1). Either the formulas and definitions had never been learned or they had been forgotten or garbled in their usage. As bad as the adults seemed to be, the 17-year-olds were worse. Let me suggest that the reason that so many people fail to solve the problem is that their knowledge and experience with area and perimeter have been only at the formula level and have left them with only a superficial understanding of the concepts. The purpose of this activity is to act as a forerunner to the development of the formulas for area and perimeter. It attempts to get the student physically involved in experiencing area and perimeter at the most fundamental level.

ACTIVITY: FOOT SIZE AREA AND PERIMETER EXPERIENCE

Materials: The materials to be used, except for the pennies and lima beans, are from the Developing Mathematical Processes K-3 Kits assembled by the Rand-McNally Company. They include squares, discs, and rings of different sizes. Substitutes can be used freely from other regular-shaped items that are available. I have used paper clips, shower tiles, triangles, straws, and trapezoids. Patterned paper (inch and centimeter graph paper, hexagon paper) is also needed. A suggested task page is shown in Figure 2.

Procedure: The students are given a blank sheet of paper, and are asked to remove one shoe and trace the outline of their foot on the piece of paper (using pencil, pen, or crayon). Each student is given one lima bean and

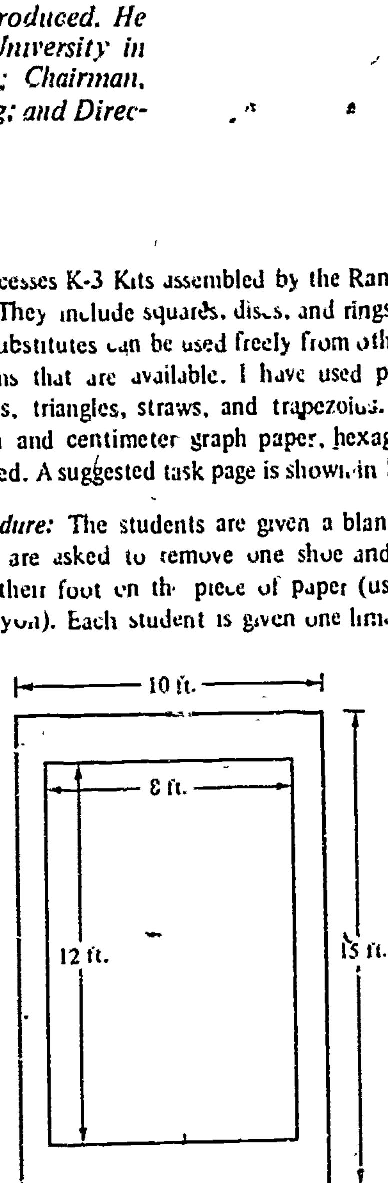


Figure 1

NONSTANDARD MEASURE
FOOT SIZE. AREA-PERIMETER

	ITEM	AREA			PERIMETER	
		Guess	Result		Guess	Result
1.	Lima Beans					
2.	Small Chips					
3.	Large Chips					
4.	Links					
5.	Small Sqs.					
6.	Large Sqs.					
7.	Pennies					
8.	Square Inches				X	X
9.	Square Centimeters				X	X
10.	Hexagons				X	X
11.	Others					

Figure 2

told to place it in the center of the outline. The task page is distributed and the student is asked to estimate the number of beans required to cover the outline of the foot. The guess will be recorded in the area guess column of the task card. The students are then asked to guess the number of beans required to cover the outline perimeter of the foot. The guess will be recorded on the task sheet in the perimeter guess column. Several pounds of beans will be brought out, and the students will be asked to see how close their estimate is to the measured results. Accuracy of measurement is of secondary importance here and should not be stressed. Expect wild excitement from the students who made a particularly close guess. Have the students repeat the activity using the other materials on the task card numbered 2 through 7. When item 8 is encountered, the students are asked to again trace their foot, only this time on the square-inch paper. Perimeter measure is not considered on this inflexible grid so it is appropriately crossed out on the task sheet. The area guess and count are then made. After item 8 is completed, the square-centimeter paper is distributed, and then the hexagon paper. The counts made using these pattern papers are usually most accurate when the students count those designs that are more than half inside the outline and ignore those that are less than half inside the outline.

After the data have been collected, the foot sizes can be ordered or the measuring instruments can be ordered. The second of these helps the students realize that the bigger the item is, the fewer it takes to cover the area.

Mathematical experiences that are implicit in the activity are counting, ordering, estimating, comparing, and verifying. After this and several other activities for internalizing the concepts of area and perimeter, the children should be at the threshold of the development of the formulas for area and perimeter.

THE NAILBOARD: A MANIPULATIVE MODEL TO DEVELOP PROBLEM-SOLVING SKILLS

by Charles P. Geer

Students can construct their own manipulatives—in this case, nailboards. This is a task intermediate through junior high school level students can complete with a minimum of teacher supervision. The nailboards can then be used to make some basic discoveries in geometry. Charles P. Geer teaches at the Nesbit School in Belmont, California

For students to really understand and enjoy mathematics, they must do mathematics. This belief has long been supported by the learning theories of Bruner, Dienes, and Piaget, as well as by the observations of perceptive classroom teachers. These teachers understand the importance of having students actively involved with physical models that demonstrate or simplify abstract mathematical concepts. They use a variety of manipulative models to help students learn mathematics in this manner. Ideally, many of these models should be made by the students. The construction of materials of this type is important because students have a greater interest and pride in materials they make themselves. Another advantage of these materials is that students may keep them and work with them long after work in the classroom is completed.

In all mathematics it is important for students to manipulate concrete models before attempting to understand the abstract symbols or detailed algorithms that are found in mathematics. This is especially true in geometry because this branch of mathematics deals with the physical world. It is only in the textbook or classroom lecture that geometric concepts become the symbolic terms students are required to learn. For this reason, it is important to attempt to return geometry to the physical world within the limitations of the elementary school classroom.

The most helpful manipulative model for achieving this is the nailboard. This board, because of its versatility, ease of construction, and high motivational interest, is a perfect manipulative aid for students in the elementary classroom. Other advantages of the nailboard are that it frees students from the static geometric diagrams found in textbooks and it permits them to conduct experiments and solve problems in an interesting manner.

It is important for students to construct their own nailboards. This can be done with a minimum of teacher supervision while allowing students to learn the many skills involved in building the board. To construct a nailboard, the following materials are needed for each student: a 10" X 10" X 5/8" piece of plywood or particle board, twenty-five 3/4" round-headed nails, and a pattern sheet containing an array of 25 dots. A hammer and a can of black spray paint are also needed for every group of two or three students. To construct the nailboard, have the students follow the steps below.

1. Sand the sides and edges of the board.
2. Spray the top with black paint and let it dry.
3. Take the pattern sheet, center it on the board, and tape in place as shown in Figure 1.
4. Hammer the nails through the dots as shown in Figure 2. Be sure that the nails are all pounded in far enough to be firm and that all are the same height.
5. Remove the pattern sheet.

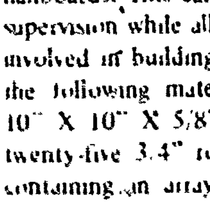


Figure 1
Pattern Sheet
with Dots - 2" apart

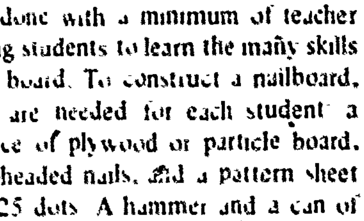


Figure 2

Each student now needs about 10 colored rubber bands and a sheet of dot paper. This paper is useful for recording the results of experiments and for preserving designs created on the nailboard. This versatile aid is now available to help students develop an understanding of the vocabulary of geometry, measure area and perimeter, and explore the properties of many geometric shapes.

The nailboard is also an excellent instrument for solving a number of fascinating problems requiring only basic geometric skills. Typically these problems develop thinking skills in a motivating manner. They are problems that provide for a variety of interesting assignments while strengthening problem-solving skills. Problems of this type often defy solution with pencil and paper but become an interesting challenge with rubber bands and a nailboard. In these situations the nailboard reaches its highest level of usefulness; suitable substitutes do not exist.

The eight problems presented here were selected because they provide a variety of problem-solving situations that lend themselves to solution on the nailboard. These eight problems are appropriate for students in grades three through eight and are listed according to difficulty.

Activity 1

The longest and shortest line segments that you can make on a 5 X 5 nailboard are shown in Figure 3. How many segments of different lengths can you make on the nailboard? Remember that two line segments are the same length if the two nails they touch are the same distance apart.

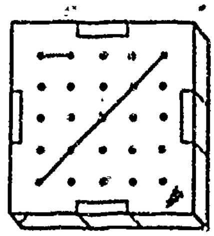


Figure 3

Activity 2

A diagonal line connects nonadjacent sides of a polygon. A square and all other quadrilaterals have a maximum of two diagonals. Make each of the polygons listed on the chart that follows, and determine the maximum number of diagonals each can have. Study your nailboard and observe the many polygon designs made by these diagonals.

Name of Polygon	Number of Sides	Number of Diagonals
Triangle	3	____
Quadrilateral	4	2
Pentagon	5	____
Hexagon	6	____
Heptagon	7	____
Octagon	8	____

Activity 3

Moving only in a horizontal or a vertical direction, find the shortest path between Points *A* and *B* on the nailboard shown in Figure 4. How many units long is this path? Are there other paths that are the same length? How many of these paths can you find?

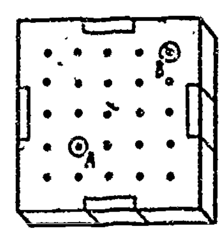


Figure 4

Activity 4

Using a 3 X 3 array of nails on your nailboard, the maximum number of sides a polygon can have is seven. Can you make this polygon? What is the maximum number of sides a polygon can have using a 4 X 4 array of nails on your nailboard? What is the maximum number of sides a polygon can have if you use all the nails on your nailboard?

Activity 5

Use only two rubber bands and your nailboard to construct—

a. Three triangles.
b. Five triangles and a pentagon.
c. Two triangles that share two line segments.

Two triangles that contain a quadrilateral and a pentagon

Activity 6

Squares having areas of 1 square unit and 4 square units are shown in Figure 5. Other squares with different areas can also be constructed on the nailboard. Complete the following table summarizing your results.

Area of Square	Possible To Make on Nailboard
1	Yes
2	____
3	____
4	Yes
5	____
6	____
7	____
8	____

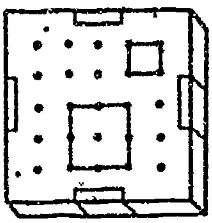


Figure 5

Activity 7

Eight triangles of different shapes, each with an area of one square unit, can be constructed on your nailboard. How many of these can you make?

Activity 8

A map of the Kingdom of Trianglevania is shown in Figure 6. King Isosceles of Trianglevania died suddenly and left the kingdom to his four sons. Each son was to receive an equal share of the kingdom and each share was to have the same shape as the original kingdom. Use your nailboard and only one more rubber band to divide up the kingdom for the four sons.

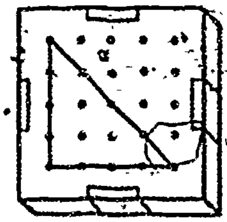


Figure 6
Kingdom of Trianglevania

These eight problems present a variety of thought-provoking activities for the elementary school student. Many of these activities can be extended by interested students who make their own discoveries while experimenting with the nailboard. After completing activities such as these, many students are ready to create and solve their own problems using a nailboard. It is hoped that the reader will be encouraged to use the nailboard to help elementary school students develop problem-solving strategies. The potential of the nailboard is unlimited for the teacher who wishes to explore this frequently neglected topic in mathematics instruction.

MULTIPLE STRIPS

by James W. Heddens

Multiple strips can be used to teach multiples, common multiples, and least common multiples as the name implies. They can also be used to explore equivalent fractions, common denominators, and addition and subtraction of unlike fractions. The author is affiliated with the Department of Elementary Education, Kent State University, Kent, Ohio.

DESCRIPTION OF TEACHING AID (Figure 1)

1. Nine multiple strips in a set.
2. Each strip has 10 multiples for a given number.

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
8	16	24	32	40	48	56	64	72	80
10	20	30	40	50	60	70	80	90	100
12	24	36	48	60	72	84	96	108	120

Figure 1

DIRECTIONS FOR CONSTRUCTING MULTIPLE STRIPS

Multiple strips can be made to look professionally prepared by using good quality colored cardboard and dry transfer lettering, and by having the material laminated. Although the set described here does not include multiple strips for 7 or 9, multiple strips for 7 and 9 may be included in your set. At least two sets of multiple strips will be needed in order to model two fractions with like denominators.

For each set of multiple strips use a brightly colored sheet of heavy cardboard which is about 25 mm long and about 13½ mm wide. Thus, each multiple strip will be 1½ mm by 25 mm. The cardboard should be separated lengthwise into 10 equal spaces. Use a black magic marker to line the cardboard. Line the width of the cardboard with spaces of 1½ mm. Use dry transfer letters and place the numerals on the multiple strips as shown in Figure 1. Laminate the multiple strips prior to cutting the strips apart. You now have a set of multiple strips for teaching eight different mathematical ideas as listed below.

YOU CAN TEACH

1. Multiples
2. Common multiples
3. Least common multiples
4. Equivalent fractions
5. Common denominators
6. Least common denominators
7. Addition of unlike fractional numbers
8. Subtraction of unlike fractional numbers

SUGGESTED TEACHING PROCEDURES

Multiples

Pass a set of nine multiple strips to each student. Have each student locate the multiple strip

1	2	3	4	5	6	7	8	9	10

and place it in front of himself or herself. Next have the student place the strip with the multiples of 2. Continue this process until all nine of the multiple strips are placed on the desk in front of each student as shown in Figure 2.

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
8	16	24	32	40	48	56	64	72	80
10	20	30	40	50	60	70	80	90	100
12	24	36	48	60	72	84	96	108	120

Figure 2

The students should study the patterns of the strips and tell the class what they can discover. They will discover such things as these: The second strip is like counting by 2's. The fifth strip is counting by 5's. The third strip has the multiplication facts for 3. Relate the multiplication facts to the multiple strips. Introduce the word *multiple* and discuss its meaning with respect to the multiple strips. Discuss the relationship of the set of whole numbers to a set of multiples. Have the students generate a set of multiples for a given number. Check their multiples with the strips. Extend the set of multiples beyond the strips. The students will need practice counting by 1's, 2's, 4's, etc.

Common Multiples

After the students understand multiples and can generate sets of multiples for a given number, then teach common multiples. Ask the students to select any two strips and look for common multiples—numbers that appear on both strips. For instance, if the student selects the 2 and 3 strips, she or he can place them side by side and then move one strip to the left or right and compare each of the numbers.

If the top strip is moved to the left, the 6's, then the 12's, and then the 18's will line up (Figure 3).

2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30

Figure 3

Each number (6, 12, and 18) is a common multiple for the pair of numbers (2, 3). Extend the common multiples beyond the strips. Use this technique with other pairs of strips until the students understand common multiples.

Least Common Multiples

Discuss with the students why the smallest lined-up number is called the least common multiple. In the example of the common multiples of 2 and 3, we see that the 6 is the least common multiple of the set 6, 12 and 18. Provide the students with many experiences locating common multiples and then with naming least common multiples. Use many different combinations of two multiple strips. Then have the students find a set of common multiples for any three strips, and then name the least common multiple. For example, what is the least common multiple for the 2, 3, and 4 strips? The students should discover that 12 is the least common multiple for 2, 3, and 4. Have the students discover least common multiples for 4 or 5 different strips at one time.

Fractions

A fraction may be represented by using the first number of a strip for the numerator and the first number of another strip for the denominator. For example have the students represent the fraction 1/5 by placing the 1 strip over the 5 strip (Figure 4). Have the students read the

1	2	3	4	5	6	7	8	9	10
5	10	15	20	25	30	35	40	45	50

Figure 4

equivalent fractions for 1/5. What ideas can the students discover about the fraction 1/5?

By having the students study the two multiple strips to construct fractions, they can discover many ideas about equivalent fractions. Through questioning, the students should develop some basic generalizations about equivalent fractions. Have them apply their new discoveries to equivalent fractions by extending the set of equivalent fractions beyond the strips.

Common Denominators

Discuss what the phrase *common denominator* means. What would common denominator mean for the fractions 1/2 and 3/4? Have the students set up the strips for the two fractions 1/2 and 3/4. Move the pairs of strips back and forth until the common denominators of 4 are together (Figure 5). Now move the 3/4 pair of strips to the right until the 8's are aligned. Move the strips so that the 12's are aligned, then align the 16's and then the 20's. The students should discover that there are many common denominators.

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20

3	6	9	12	15	18	21	24		30
4	8	12	16	20	24	28	32	36	40

Figure 5

As soon as they can move the strips to locate many common denominators, then introduce the phrase *least common denominator*. Discuss the meaning of the word *least*. What would least common denominator mean? Now set up many examples and manipulate the strips to find a set of common denominators, then locate, by aligning the strips, the least common denominator for each pair of fractions. Provide three different fractions and have them use the strips to discover a set of common denominators and then state the least common denominator. For example, use the fractions 3/4, 1/8, and 2/5. Provide many experiences and then see if the students can generalize a method for locating the least common denominator without using the strips.

Addition and Subtraction of Fractional Numbers

After the students have generalized a technique for addition and subtraction of fractional numbers with like denominators, they should be ready to add and subtract fractional numbers with unlike denominators. Because of using the strips, the students should understand least common denominator. They should also know how to add fractional numbers with like denominators. Using these two techniques there is really nothing new to teach. Provide a real situation and the discovery technique, and the students should discover and generalize a technique for addition and subtraction of fractional numbers with unlike denominators

4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50

3	6	9	12	15	18	21	24	27	30
8	16	24	32	40	48	56	64	72	80

The least common denominator is 40.

The common denominators of 40 are placed one above the other

The numerators are added.

Figure 6

Add $\frac{4}{5}$ and $\frac{3}{8}$ (Figure 6).

$\frac{4}{5} + \frac{3}{8} = ?$

The least common denominator is 40.

$\frac{32}{40} + \frac{15}{40} = ?$

The numerators are added.

$\frac{47}{40} = ?$

2	4	6	8	10	12	14	16	18	20			
3	6	9	12	15	18	21	24	27	30			
			5	10	15	20	25	30	35	40	45	50
			12	24	36	48	60	72	84	96	108	120

Place common denominators together.

Subtract numerators (8-5)

Figure 7

Now solve: $\frac{2}{3} - \frac{5}{12} = ?$

Find fraction strips for $\frac{2}{3}$ and $\frac{5}{12}$ (Figure 7).

The least common denominator is 12.

$\frac{8}{12} - \frac{5}{12} = ?$

The numerators are subtracted.

$\frac{3}{12} = ?$

THE SURFACE AREA AND VOLUME OF THE SPHERE

by Mary Laycock

Mary Laycock is a mathematics specialist at the Nueva Day School and Learning Center in Hillsborough, California. Her premise is that students should understand where formulas come from and why they work. The activities here are intended for bright sixth graders or any junior high school through tenth grade mathematics students.

Prerequisite Understandings

1. Why is the area of a circle equal to pi times the radius squared (Figure 1)?

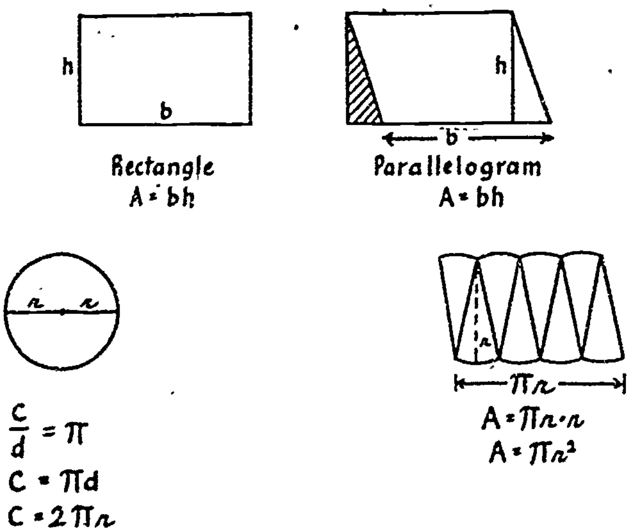


Figure 1

2. Why is the volume of a pyramid equal to one-third the length of the base times the altitude? The best demonstration uses the three pyramids from the ESS Geoblocks which fit together to form a cube. The pyramids are congruent, and the base and height of the pyramid are the same as the base and height of the cube (Figure 2).

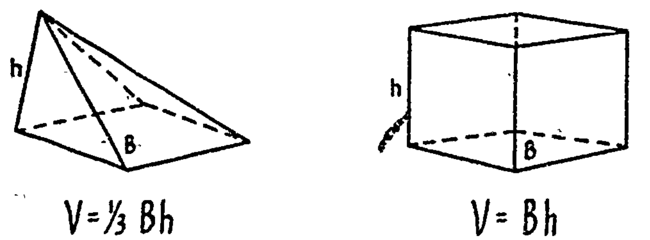


Figure 2

THE SURFACE AREA OF THE SPHERE

Materials needed: Two styrofoam spheres (7", least expensive), two small nails, two long pieces of yarn of the heavy variety used to wrap decorative packages.

Directions: Cut one of the spheres in half; a coping saw does this acceptably. Cut the other one into eight congruent pieces; each cut should go through the center of the sphere. (See Figure 3.)

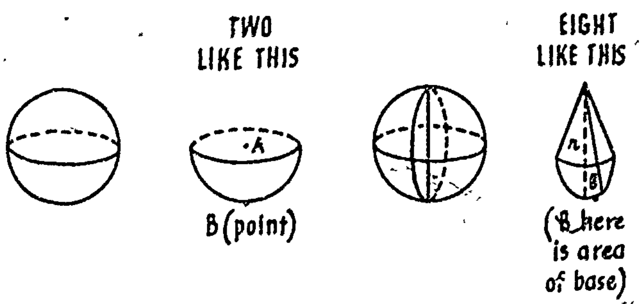


Figure 3

Place a nail at A and wrap the yarn carefully to cover the great circle for which A is the center. Tie a knot or cut the yarn when the circle is covered.

Place another nail at B (the apparent South Pole) and wrap the hemisphere with yarn. Knot or cut at this point.

Compare the two pieces of yarn. The one starting at B will be approximately twice the length of the one starting at A.

Conclusion: The area of the great circle:

$$A = \pi r^2$$

The area of a hemisphere:

$$A = 2\pi r^2$$

The total area of the sphere:

$$A = 4\pi r^2$$

THE VOLUME OF THE SPHERE

Consider the eight pieces of the sphere. Each resembles a pyramid. Make sure the learners recall the relationship between a pyramid and a solid with equal base area and altitude.

The volume of the sphere can be written.

$$V = \tfrac{1}{3} r B_1 + \tfrac{1}{3} r B_2 + \tfrac{1}{3} r B_3 + \tfrac{1}{3} r B_4 + \ldots + \tfrac{1}{3} r B_8$$

$$V = \tfrac{1}{3} r \left(B_1 + B_2 + B_3 + B_4 + \ldots + B_8\right)$$

$$\text{Since } B_1 + B_2 + B_3 + B_4 + \ldots + B_8 = 4\pi r^2$$

$$V = \tfrac{1}{3} r \cdot 4\pi r^2 = \tfrac{4}{3}\pi r^3$$

A PERCENTAGE VISUALIZER

by M. Stoessel Wahl

The percentage visualizer is a unique manipulative to teach percentage to upper elementary through secondary level students. According to the author, an Associate Professor of Mathematics at Western Connecticut State College, adults often ask, "Why didn't they show me that when I was learning?"

Percentage problems, being needed in everyday life, have survived in the "back to basics" movement. The hand-held calculator has come up with a simple push-button solution to these percentage problems. However, concerned mathematics teachers hope to give their young learners a basic understanding of the meaning of percentage. With this objective in mind, the author wishes to share with other teachers the way to construct and use a teacher-made manipulative d vice that helps students to understand just what percentage is all about.

A class of sharp fourth graders who were discussing their current interest in basketball free throws was presented with the following problem. Jimmy managed to achieve 13 free throws in 17 attempts and Pat achieved 12 free throws out of 15 attempts. Just as José had made 9 free throws in 11 attempts, the bell rang and that finished their game. How can we decide which one of the three was the most successful? The planned raggedness of the scoring results presented some problems to the young learners and a lively discussion followed. They soon decided as a class that they needed some consistent standard to judge by. With some indirect but guarded suggestions from the teacher, they decided to use a standard – "by hundreds" numbers–for their measure.

Each score was then converted, with the aid of the teacher, using a rate pair or proportional approach, to "by hundreds" numbers. Jimmy's 13 out of 17 turned out to be *76* "by hundreds", Pat's 12 out of 15 was *80* "by hundreds", and José's 9 out of 11 shots was *82* "by hundreds." Therefore, José won.

Later, after the class became proficient in using the standard of "by hundreds" numbers, they were let in on the secret that the word for *by* in Latin is *per* and the Latin word for *hundred* is *cent* so they could, if they wished, call their "by hundreds" numbers "percents."

The solution of the proportions was based on the fact that if two ratios are equal, their cross products are equal. The use of this technique, when three variables are given and a box placeholder is used for the unknown quantity, enables the student to handle all three cases of percentage or "by hundreds" problems.

To some pupils this was merely an experience with paper-and-pencil math, and the results were not too well understood. Hence, a teacher-made gadget was developed that convinced most of the young learners of the true meaning of percentage or "by hundreds" numbers.

The manipulative device consisted of a large aluminum sheet with enough equally spaced holes that a square array of 100 holes by 100 holes could be set off by narrow masking tape strips. The horizontal holes were labeled PARTS and the vertical holes were labeled WHOLES. Pegs of a proper size to fit into the holes were secured to help in the visualization of the problem. In the upper lefthand corner a gold colored cord was inserted. This was attached to a plumb bob.

Let us visualize a simpler problem with a solution that can be obvious to the learner. Assume a score of 34 free throws out of 50 attempts. What is the "by hundreds" number? A peg is inserted on the horizontal line at 34 for the *part*. For the *whole* a different colored peg is inserted on the vertical line at 50. Now, using the inserted pegs as a guide, a third peg of still another color is located at a point 34 over and 50 down. This represents the position of 34 out of 50. The gadget can now be tipped so that the plumb bob of the line centered at (0,0) passes over the third peg. This line, pulled straight by the force of gravity, cuts the lower "by hundreds" line at 68. Place a fourth peg in that position. Hence 34/50 equals 68 "by hundreds" or 68 percent.

The students soon discover that the device can be

Percentage Visualizer

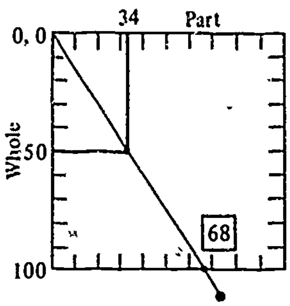


Figure 1

used in the same way to read "by sixties" or "by nineties" numbers as well. However, they can be led to prefer the "by hundreds" numbers because the problems can easily be related to our money system based on a whole of one dollar or 100 cents.

This manipulative device can also be used to visualize the other two kinds of percentage problems. For instance, what number is 68 percent of 50? Using pegs attached to stretchable rubber cord, stretch a line from 50 on the right vertical to 50 on the left vertical. At each point on the rubber line the vertical reading is now 50. Place another colored peg on the bottom horizontal line at 68. Now, carefully stretch the gold plumb bob line from its (0,0) point through this peg-marked 68 point. Place a peg where the rubber line and the gold line intersect. Read this value on the horizontal line. It is 34. Hence, it should be true that 34/50 = 68/100. Encourage the students to check this by multiplying the cross products. This gives a great deal of hidden drill practice in multiplication for the students.

Percentage Visualizer

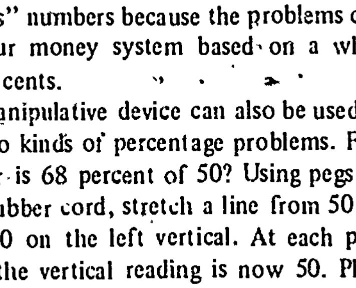


Figure 2

The solution of the last case in percentage is similar in approach. 34 is 68 percent of what number? This time attach the rubber band to pegs on the top and bottom at 34 for each. Now each point on the rubber line has a reading of 34. As before, place a peg at 68 on the bottom line, and stretch the gold line through it. Place another colored peg at the intersection of the rubber and gold lines. Read its vertical value. 50? You are right, but you will want your pupils to check it by cross multiplication.

Percentage Visualizer

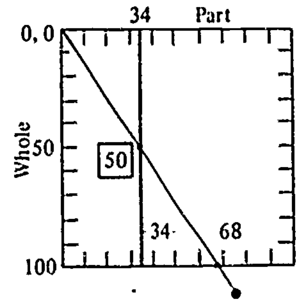


Figure 3

MATHEMATICAL NOTES FOR THE TEACHER

And what to do when the bright student asks you why it works? First of all we must remember some geometric theorems that underlie the situation. Remember? Two right triangles are similar if an acute angle of one is equal to an acute angle of the other. Since the triangles we analyze are both right triangles, and since they share the same acute angle, they are, therefore, similar triangles. Also, corresponding sides of similar triangles are proportional (their ratios are equal). Therefore, the part of the smaller triangle is to its whole as the corresponding part of the larger triangle is to its whole (a hundred)–i.e., 34/50 = 68/100 or 34/50 = 68%. It is advisable that problems, once illustrated by the device, be solved with pencil and paper by the student so that the student can see the relationship between the two.

Perhaps it is also wise to illustrate use of the place holder in the three problems:

$$\frac{34}{50} = \frac{\square}{100} \qquad \frac{34}{\square} = \frac{68}{100} \qquad \frac{\square}{50} = \frac{68}{100}$$

Cross multiply:

$50 \square = 34 \times 100$
$\square = 68$

$68 \square = 34 \times 100$
$\square = 50$

$100 \square = 50 \times 68$
$\square = 34$

MAKING THE PERCENTAGE VISUALIZER

This gadget was made possible by discovering in a hardware store a set of large, metre-square sheets of aluminum for various building purposes. One of these, used for ventilation, has an array of 125 holes by 125 holes. The cost is in the vicinity of $5.00. Narrow black masking tape was used to block off an array of 100 holes by 100 holes. Holes were punched in the black tape. For easier reading of numbers, the border parts were marked off in units of tens. (See Figure 4.) Small, multicolored pegs were found that fit the holes. The gold cord used with the plumb bob was leftover Christmas package wrapping. The elastic thread was purchased at the dime store. The plumb bob is a fisherman's sinker. The construction is quite simple in relation to the understanding the student gains from its use, for it is truly a percentage visualizer.

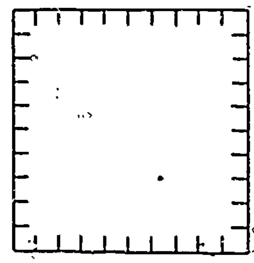


Figure 4

CIRCULAR GEOBOARD TASKS

by Murray Rudisill and Curtiss Wall

The following is a collection of tasks for secondary level students using geoboards. The concepts covered are proof that manipulatives are not just for the primary grades. Murray Rudisill and Curtiss Wall are both associated with Old Dominion University, Norfolk, Virginia, in the field of mathematics education.

Materials: Rubber bands, string, protractor, ruler, the circular geoboard pictured in Figure 2.

A *circle* is a plane closed curve. Therefore, it separates the plane into three subsets: the boundary or circumference of the circle, the interior of the circle, and the exterior of the circle (Figure 1). The geoboard you will be using is called a *circular geoboard*, and it has a nail in the interior of the circle called the center (Figure 2).

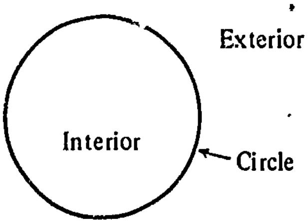


Figure 1

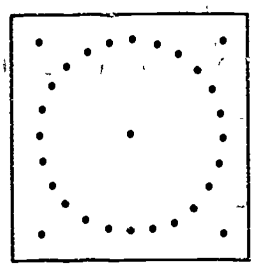


Figure 2

Task 1

Connect the center of the circle with nail A on the circumference of the circle (Figure 3). This line segment is called the *radius (r)*. Use your ruler to measure the radius OA. ______

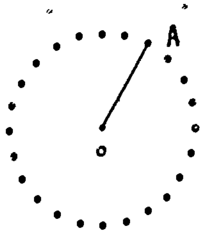


Figure 3

Connect the center of the circle with any other nail on the circumference of the circle. Measure the radius.

______

What can you conclude about the distance between the center of the circle and any point on the circumference?

______

Make some radii (more than one radius) on your geoboard. What is the intersection of the radii? ______

Task 2

We can connect a rubber band to two nails on the circle so that it passes through the center of the circle. This is called a *diameter (d)* of the circle. Connect nails to make four diameters in different directions. Measure the lengths of these diameters. Are these lengths the same or are they greater or less than each other? ______

The measure of the diameter is considered the width of the circle. What can be said about the width of the circle from the previous example? ______

Task 3

Use your ruler to measure the length of one of your diameters. Compare your result with the length of a radius recorded in task 1.

Which of these rules work(s)? ______

a. $r = d + 2$

b. $r = d - 1$

c. $d = 2r$
d. $d = r + r$

If the diameter is 10 inches, the radius would be ______ inches.

Task 4

Use a rubber band to connect any two nails on the circumference of the circle. This line segment is called a *chord*. A chord begins on the circumference of the circle and ends on the circumference of the circle.

Make some chords on your geoboard (Figure 4).

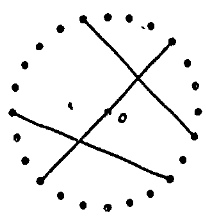


Figure 4

Look at the shortest chord you can make. How many of these chords can you make? ______

What is the name of the longest chord you can make? ______

How many of these chords can you make? ______

Task 5

A chord cuts off a portion of the circumference of the circle called an *arc*.

Put your finger on point A. Trace along the circumference of the circle until you come to point B. This is called arc AB (Figure 5).

Form some arcs on your geoboard.

Figure 5

Task 6

Choose a nail in the exterior region. Stretch a rubber band from this nail through the interior to a nail on the boundary. This line is called a *secant*.

Connect two adjacent exterior nails. Notice that the line formed barely touches one nail on the boundary of the circle. This line is said to be tangent to the circle and is, therefore, called a *tangent* line.

Practice making some secant and tangent lines (Figure 6).

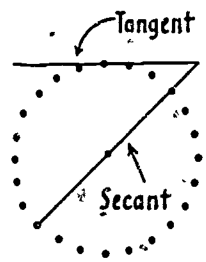


Figure 6

Task 7

Stretch a rubber band forming arc AB to the center of the circle (Figure 7). The angle formed is said to be *subtended* by arc AB. The angle is also called a *central angle*.

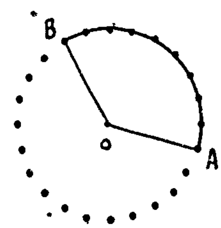


Figure 7

A central angle is an angle formed by two radii of a circle. What two radii form the central angle subtended by arc AB? ______

Arc AB and central angle AOB are measured in degrees. Use a protractor to measure angle AOB. The number of degrees in angle AOB will be the number of degrees in arc AB.

Task 8

On your geoboard, make the central angle subtended by arc *CD* (Figure 8).

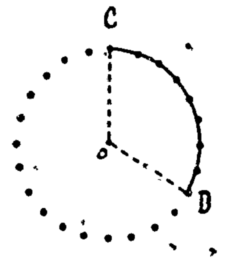


Figure 8

What two radii form this central angle? ________
What is the degree measure of this angle? ________
What is the degree measure of arc *CD*? ________

Task 9

Make the smallest central angle you can form on the geoboard. It measures ________ degrees.

Each central angle formed by connecting two nails that are next to each other with the center equals 15 degrees.

What do you think a central angle like angle *E* in Figure 9 will measure? ________ Check by measuring:

Angle F = ________ degrees

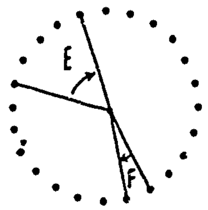


Figure 9

Which formula below gives us the number of degrees in our central angle for this circle? ________ ($\triangle$ stands for the number of nails *between* the two touched on the circumference; C stands for the number of degrees in the central angle.)

a. $C = (\triangle)(15)$
b. $C = (\triangle + 1)(15)$

Task 10

Form a central angle on the geoboard like the one in Figure 10 (the angle formed is indicated by the arrow).

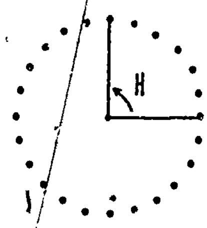


Figure 10

Use your protractor to measure the central angle you formed: it is ________ degrees.

Task 11

Without lifting your finger, trace arc *BC*, arc *CD*, arc *DE*, and arc *EB*. Did you trace around the entire circumference of the circle? ________

What two radii form the central angle subtended by arc *CD*? Arc *DE*? Arc *EB*? Arc *BC*?

Make these angles on your geoboard and find the degree measure of each.

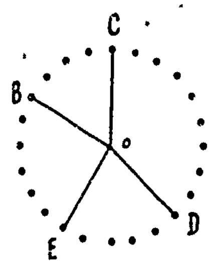


Figure 11

We can conclude that—

a. The sum of the central angles of any circle equals ________.

b. A circle contains ________ ________.

Task 12

The part of the circle cut out by a central angle is called a *sector* of the circle. A sector is a figure made by two radii and the arcs between their endpoints (Figure 12).

Make some sectors on your geoboard. How many different sized sectors can you make? ______

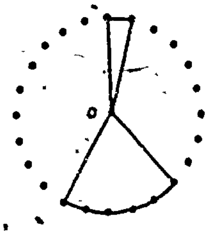


Figure 12

Task 13

Divide the circle into four equal sectors.

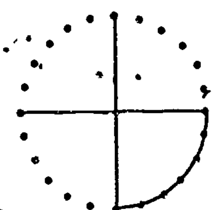


Figure 13

What do you think each of the central angles or arcs will measure? ______ Check your answer using a protractor.

Task 14

Divide the circle into two equal sectors (Figure 14). These sectors are called *semicircles (semi* means "half")

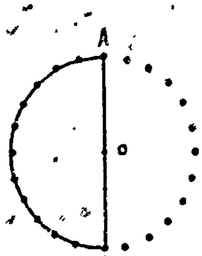


Figure 14

What do you think each of the central angles or arcs will measure? ______ What chord divides the circle into two equal parts? ______

Task 15

Make any two chords on your geoboard so that their point of intersection is a point on the circumference of the circle (Figure 15). This angle is an *inscribed angle.*

Make some inscribed angles on your geoboard.

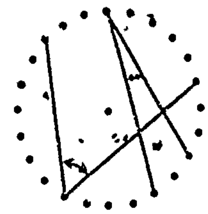


Figure 15

Task 16

Angle B in Figure 16 is an inscribed angle. Find its degree measure. ______

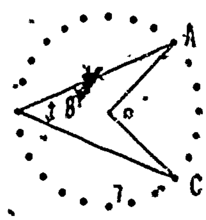


Figure 16

Make the central angle subtended by arc AC. Find its degree measure. ______

Compare the degree measures of the inscribed angle and the central angle.

Predict what the inscribed angle formed by connecting the angles on the circumference touched by the sides of a 120-degree angle will measure. ______

Which of these rules works? ______

a. Inscribed angle = central angle.
b. Inscribed angle = twice the central angle
c. Inscribed angle = half the central angle

Task 17

Form an inscribed angle by connecting the end points of the diameter with any other nail on the circumference (Figure 17). Measure the inscribed angle you just formed. It is ______ degrees.

Now move the rubber band to any other nail on the circumference, leaving the sides of your inscribed angle

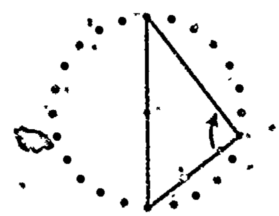


Figure 17

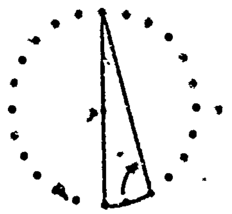


Figure 18

touching the ends of the diameter (Figure 18). Measure this new angle. Is it the same size as or different from the other inscribed angle you formed? ____________ If you move the vertex of this inscribed angle to another nail, will the measure of your inscribed angle change? ____________

Indicate whether each of these statements is true or false:

a. Any angle inscribed in a semicircle measures 90 degrees. ____________
b. Two angles inscribed in a semicircle might measure a different number of degrees. ____________
c. If three angles inscribed in a semicircle have their vertices each touching a different nail, their measure in degrees will still be the same ____________

Task 18

Form any three-sided figure by connecting three nails on the circumference of the circle. This three-sided figure is called a *triangle*. Measure (or calculate, using inscribed angles) each of the three angles inside your triangle. What do these angles add up to? ____________

Task 19

Form any four-sided figure by connecting four nails on the circumference of the circle (the sides are not allowed to cross each other), an example is shown in Figure 19.

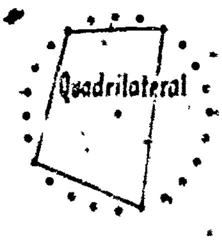


Figure 19

We call such a figure a *quadrilateral*. Notice that the sides are straight line segments that touch only at the four vertices.

Task 20

Measure (or calculate) the four angles inside your quadrilateral (remember that these are inscribed angles): each will be one-half of the central angle formed when you connect the end points of the angle to the center of your circle. The formula for calculating the central angle is given in Task 16.

What do the inside angles of your quadrilateral add up to? ____________

Task 21

Form a five-sided figure by connecting five nails on the circumference of the circle; this figure is called a *pentagon* (Figure 20). The inside angles of the pentagon add up to ____________ degrees.

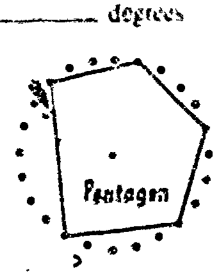


Figure 20

Complete

Number of Sides	Name	Sum of Interior Angles
three	triangle	____________ degrees
four	quadrilateral	____________ degrees
five	pentagon	____________ degrees

Which rule would give you the sum of the interior angles of these figures (N = number of sides, S = sum of interior angles)?

a. $S = (N - 2)(180)$
b. $S = (N + 2)(180)$

Task 22

Let's take a closer look at triangles. There are three types of triangles: *Equilateral* triangles have three equal sides; *scalene* triangles have no equal sides; *isosceles* triangles have two equal sides.

Form an equilateral triangle.

How many degrees does each of the interior angles measure? ______________

Form an isosceles triangle.

How many degrees does each of the interior angles measure? ______________

Write *isosceles*, *equilateral*, and *scalene* in the correct blanks.

Type of Triangle	Number of Angles Equal to Each Other
____________	three
____________	two
____________	none

Task 23

Construct any type of triangle on the geoboard. Measure each angle and find out what angle has the greatest measurement. Now measure the sides of the triangle and find out which side is the longest. What can be said about the position of the longest side of a triangle in relation to the largest angle? ______________________________

PENCIL AND PAPER ARE YOUR MANIPULATIVE EXTENDERS

by John Van de Walle

In the author's words, this article "speaks directly to making a transition from physical materials to symbolism." This is an area often neglected in dealing with manipulatives. John Van de Walle is an Assistant Professor of Education at Virginia Commonwealth University, Richmond

Today no one doubts that manipulatives are the very best means of teaching mathematical concepts to students. Piaget and those who have tested his theories in classrooms have convinced us that during their elementary school years students learn initially through concrete or manipulative experiences. Through workshops, coursework, and publications, teachers are constantly encouraged to use counters, place value pieces, rods, and many other materials to either replace or augment a textbook instructional mode. This emphasis on manipulatives is accepted as a valid and proper direction for the teacher of elementary school mathematics.

On the other side of this coin, however, are a number of facts that give us pause when considering a total manipulative approach. First among these facts is that the end goal of a mathematics curriculum is almost totally symbolic. We want—we *expect*—that students will eventually work with, use, and interact with various arrangements of numerals and other mathematical symbols such as operation signs, parentheses, and equal signs. There seems to be a definite gap between this symbolic form of mathematical dexterity and the desirable manipulative methods of good mathematics instruction. The textbooks we use frequently employ pictures of manipulatives on the "instructional" pages. However, we find that students rarely attach much significance to these pictures. Generally they are ignored. At any rate the texts quickly move on to symbolic drill exercises anyway. This gap between manipulatives and symbols must be bridged.

And there are still other difficulties with manipulatives. Almost never are there enough to go around. Elaborate arrangements for learning stations and classroom management schemes must be employed. Some manipulative modes are slow and tedious. And how do we wean students away from them?

There is a way to bridge the manipulative-symbolic gap and gain some added benefits at the same time. *By no means should we abandon our efforts with hands-on materials.* On the contrary, we should take special care to use manipulatives properly. Beyond this, however, let us teach students to draw pictures *of their own* to show what they are doing (or would do) with the manipulatives. Students can easily be taught simple drawing techniques that can be associated with the most important manipulatives we use in the classroom.

With the aid of pencil and paper extensions (drawings) of the manipulative mode, instruction moves deliberately in graduated steps from the concrete to the symbolic.

Step 1. *Strictly manipulative.* The student models work with materials. Answers to problems may be recorded, but all work is done manipulatively.

Step 2. *Manipulative-drawing.* The student proceeds as in step 1 but is taught to draw pictures of his/her work as it is completed. Drawings go right on the worksheets along with the problems and answers.

Step 3. *Drawings only.* The student draws the now familiar manipulatives but does not have the actual materials present. By now the drawings must be meaningful and easily drawn. Drawings still accompany all work.

Step 4. *Symbolic.* Work is presented and solved without the use of drawings or manipulatives. The student will enter this stage at his/her own rate.

This deceptively simple technique requires very careful instruction. Students must be taught *how* to make the drawings. The association with the manipulative must be clear to the students making the drawings. Finally, they must be encouraged to use drawings frequently.

In the drawings that follow, the most commonly used manipulatives are paired with suggested drawing techniques. If these drawings are explicitly taught, the gap between manipulatives and symbols can be narrowed. Furthermore, since drawings require nothing more than blank newsprint (use text or chalkboard problems), students can eventually have their own "manipulatives" in their drawings. They will learn to "manipulate their drawings" faster than using the actual material. Even textbook pictures begin to have some meaning.

SUGGESTED PENCIL-AND-PAPER EXTENSIONS

In most cases, drawings should be done on cheap newsprint. Young students will draw freely if permitted to draw large. Newsprint is still inexpensive. Avoid forcing drawings in cramped worksheet spaces.

1. Counters of all sorts are used for various purposes from one-to-one matching activities in kindergarten to multiplication and division concepts in grades three and four to fraction concepts in grades five and six. While the drawing of counters is very easy, it must be encouraged to ensure that it is done. Figure 1 provides some examples of drawings of counters.

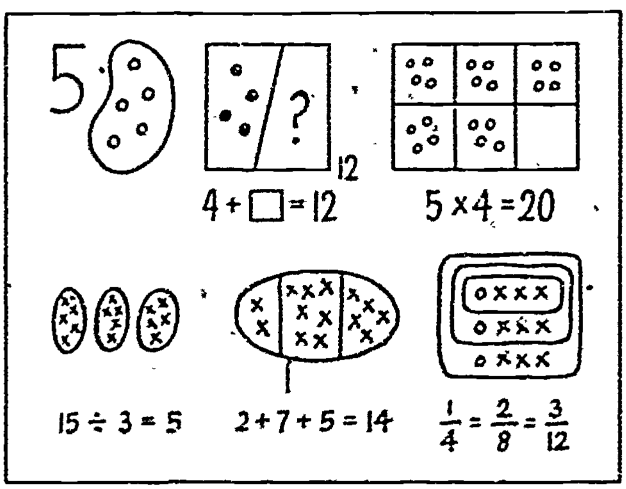


Figure 1

2. Place value materials comprise probably the single most important set of manipulatives in the elementary mathematics curriculum. The most common teacher-made versions are sticks and bundles of 10 beans and beansticks with 100 shown as a "raft" of 10 beansticks, and strips and squares cut from posterboard. The latter may use a 1 cm square for ones, a 1 × 10 cm strip for tens, and a 10 × 10 cm square for hundreds. Excellent wooden and plastic versions are also commercially available. All of these can be modeled by simple stylized drawings as indicated in Figure 2. Drawings of place value pieces can be close to actual size or as small as the students are capable of drawing. They also can be drawn in a two- or three-column place value chart in order to work addition and subtraction problems or to promote the right-left order of ones, tens, and hundreds.

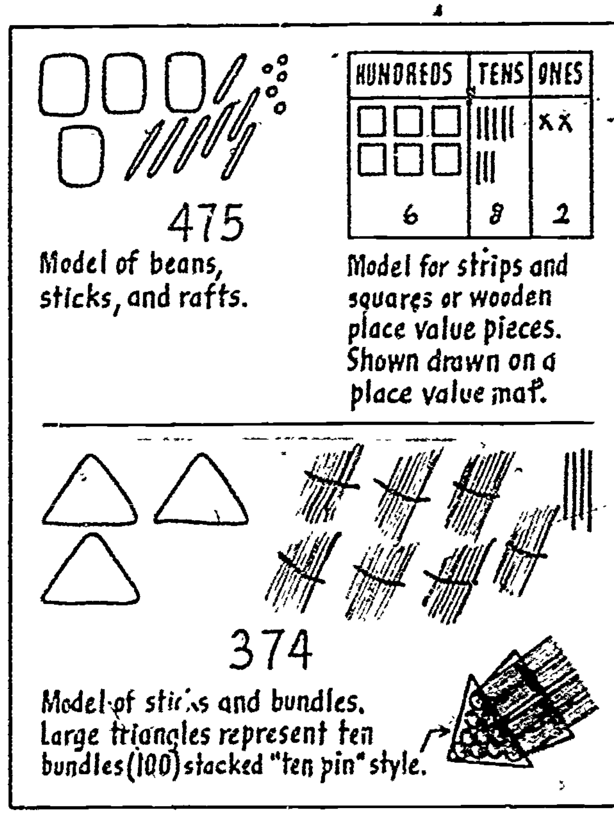


Figure 2

3. The equal arm or mathematical balance is an excellent device for all four operations. It is especially useful for emphasizing the concept of equality and for dealing with missing addends. Figure 3 illustrates a balance along with several drawings. Even if only one balance is available in the room, the drawings provide a useful extension.

4. Wooden cubes are useful as a special type of counter because they can be lined up as in a bar graph or arranged in patterns and arrays. Grid paper and crayons provide an excellent extension. Use a large grid (2 to 3 cm) for primary grades and a smaller grid (1 cm) for older students.

5. When sufficient place value materials are not available, colored counters are frequently used on place value

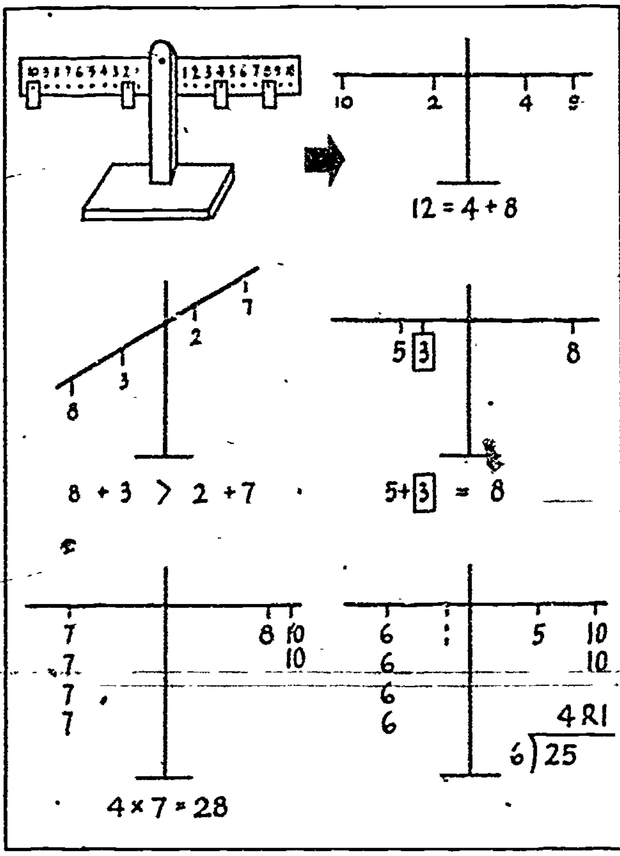


Figure 3

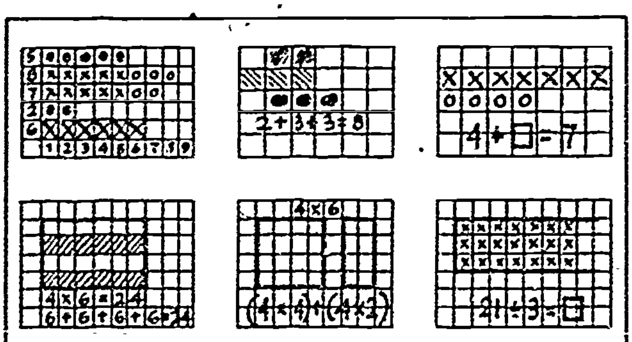


Figure 4

charts. Rather than drawing counters, numbers can be written in dittoed charts to aid in the transition. If the actual place value mats are laminated, the numerals can first be drawn directly on the mats next to the counters or place value pieces.

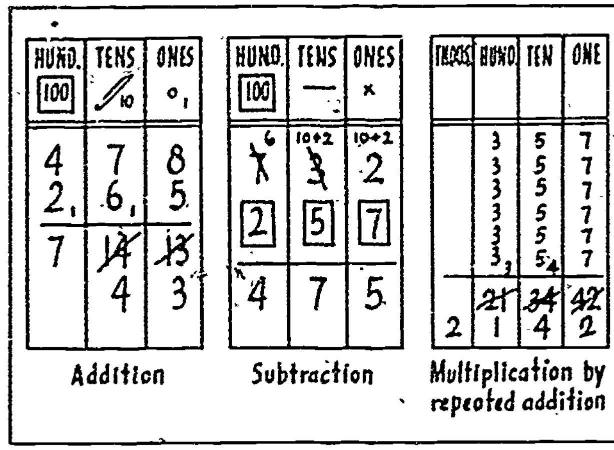


Figure 5

SUGGESTED DRAWINGS WITHOUT MANIPULATIVES

Many of the same benefits are derived when simple, meaningful drawings are encouraged even when no physical model is used. Some examples of such drawings are illustrated below.

1. When the long multiplication process is developed, one method is to use rectangular arrays partitioned between 10's and 1's. The different sections of the rectangle provide each of the partial products. Figure 6a shows a one by two-digit problem, and Figure 6b shows a two- by two-digit problem using this method of explanation. (The method used in Figure 6 is explained in the 1978 NCTM Yearbook: *Developing Computational Skills.* Reston, Va.: National Council of Teachers of Mathematics, 1978.)

2. Equivalent fractions can be developed by "slicing" squares in two directions. The drawings, which frequently appear in textbooks, can easily be drawn by students.

3. Similar drawings are often used to illustrate multiplication of two fractions. If these are used in the assigned textbook, students should be encouraged to draw their own models as well.

A word of caution about fraction drawings. It is difficult for students to divide a rectangle into equal sections. As long as they realize that the parts *should* be equal, there is no reason to be concerned about lopsided sections. On the other hand, it would be unwise to use students' drawings to compare, for example, the relative sizes of 2/3 and 3/4. In situations such as these, accuracy is essential or errors may result. Sections of circles as in "pie pieces" are

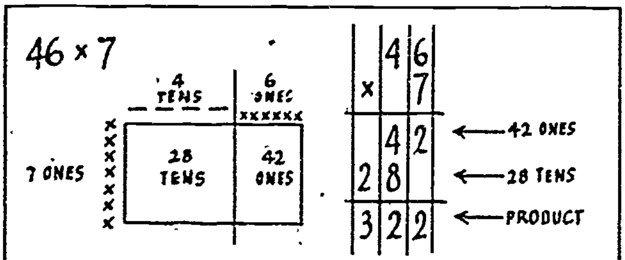


Figure 6a

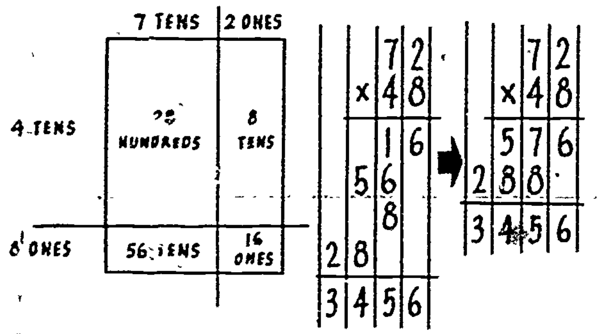


Figure 6b

much more difficult for students to draw and probably should not be attempted.

4. One useful model for decimal fractions is a prepared sheet of small 10 by 10 grids. If the area of a full square is taken as 1, then decimal parts to two places can be shaded by the students to give meaning to decimals (Figure 8). Very finely grided graph paper can be used in a similar way to investigate decimals to four places.

5. While number lines are hardly innovative, we can encourage students to make quick sketches to assist with the concepts of large numbers, decimals, and percentages.

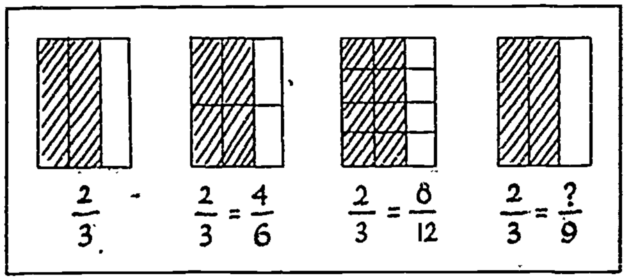


Figure 7

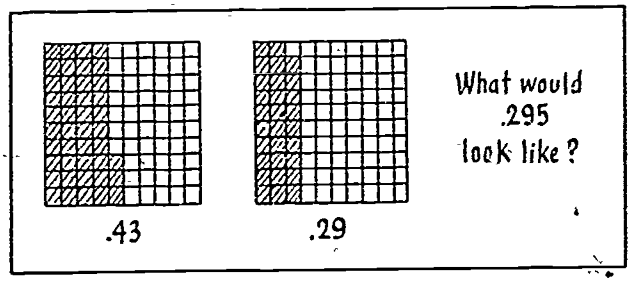


Figure 8

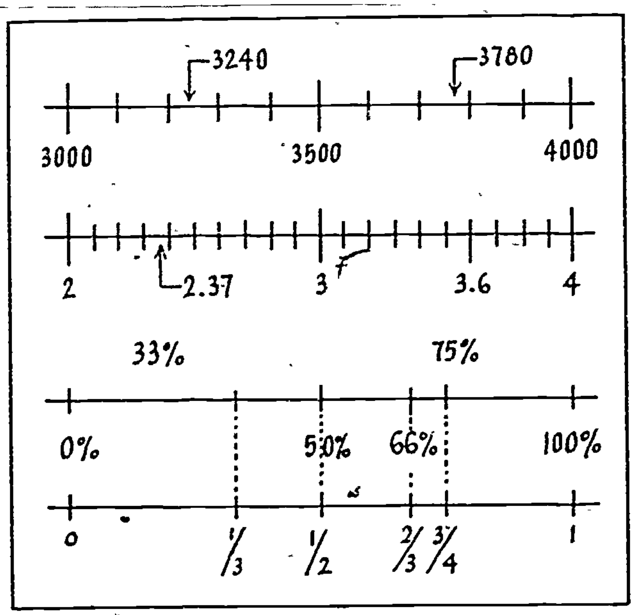


Figure 9

SUMMARY

There is a definite need to help students make the transition from manipulative materials to pencil-and-paper forms. If some time is spent deliberately teaching and encouraging them to make drawings to accompany their written work, this transition can be greatly facilitated. The pencil-and-paper "extenders" suggested here have a number of desirable attributes as teaching aids

1. They are almost cost free.
2. They usually require only blank newsprint.
3. They fill in for limited supplies of materials.
4. Students always have them handy.

As we develop in students the habit of sketching a quick drawing to go with their work, we provide them with a valuable tool for dealing with mathematics. Drawing pictures not only moves one from concrete to symbolic thought but also is, in and of itself, a valuable strategy in "learning to learn." In terms of cost effectiveness, pencil-and-paper drawings should not be overlooked as a form of "manipulative" in our classrooms.

PART II

Games

GAMES IN THE MATHEMATICS CLASSROOM

by Donald L. Zalewski

This article supports the use of games and gives many valuable suggestions for selecting and using them. It serves as an introduction to all the games that follow. The author is an Assistant Professor of Education at the University of Nebraska at Omaha. He has spoken on the subject of games at regional and national math meetings.

Anyone who watches children play a game quickly realizes that such an activity combines all the features of an ideal learning situation. The participants are intrinsically motivated to use their mental and/or physical skills to achieve a goal that they feel is important. The amusement, pleasure, and group interaction which develop in the pursuit and achievement of the goal add to the value of games as a learning activity.

In the mathematics classroom, games can be particularly effective for basic skills practice, applications, and logic and strategy development. Physical involvement games such as throwing beanbags can also help develop physical coordination and visual perception. Finally, games provide a pleasant diversion from the usual mathematics classroom "fill and drill" routine. If properly selected and used, they can help improve the attitudes of students who too often view mathematics as a dull, tedious subject.

As you select a game, you logically must consider (1) the student's ability and (2) the learning that you wish to promote. A game that is too complex to understand or to play is worthless. A game that does not involve any math skills or concepts might be fun, but it is a waste of precious time.

A third consideration in selecting a game is the type of participation you desire. Team games such as "math-downs" can involve the whole class well. However, a "one-on-one" game with dice or an individual game such as "Connect the Dots" is appropriate for small groups, math labs, and free-time activities.

Closely allied with participation is the type of competition that a game promotes. Competitive games, whether team or individual efforts, usually produce winners and losers or infer that some students are better than their peers. Competition has its merits, but you must try to give all students an equal chance to win. Skill based games favor the smartest and the fastest students, thus producing the same losers regularly. Matching opponents by ability or using games of chance (i.e., cards, dice) can help you minimize the problem.

In order to avoid the "winner" or "loser" designations, you might consider using games in which the student competes against a standard as in golf or works toward a predetermined goal as in code breaking. In these games, personal improvement or attainment of the goal is the measure of success. Noncompetitive activities such as building models (with tinkertoys, for example) or having group guessing games are recommended for students who are unable to compete well and for students who by nature are unable to accept defeat.

A fifth consideration in selecting a game is the maturity of the students. Kindergarten and early elementary students need to participate in some way at all times. As students mature, they learn to patiently wait their turns. However, a game that has all participants active by making a play or reacting to another's play maximizes the practice that games should give.

The final characteristic to check when selecting a game is feedback. Mathematics games that emphasize skills and applications necessitate regular, immediate feedback to assure that students are practicing correct responses. The teacher and an answer key are the most dependable feedback sources for young students. However, having the participants check each other's plays and responses (especially if they can profit by catching an error) provides feedback while increasing participation and motivation.

The value of games, however carefully selected, depends on how well you use them. The necessary materials must be prepared in advance. Directions, whether oral or written, must be clear to the students. If proper rule following is critical, then you might have to give examples before play starts, and you must regularly check the student's progress. Follow-up activities are important to focus on the strategies, results, and learning brought about by the game. Finally, you need to use the students' reactions and learning to evaluate the game.

Several commercial games can be used in the mathematics classroom. However, homemade games or those found in educational publications serve equally well. The main purpose here is to encourage you to effectively select and use games to supplement your mathematics teaching.

GAMES AND ACTIVITIES WITH NUMBER CUBES

by Donald L. Zalewski

Number cubes (or dice) are inexpensive additions to the mathematics classroom. This series of dice games was designed for Kindergarten- through eighth-grade-level students.

"Fill'er Up"

Equipment: One die, a supply of dry beans, corn kernels, or other markers, a number line made of squares with numbers from 1 to 25 (or further) for each participant (Figure 1).

1	2	3	. . .	25

Figure 1

Players: Any number, but two to four playing together works best. No one sits too long waiting for his/her turn. Grades K-1.

Skills:
1. Concrete one-to-one correspondence
2. Pattern recognition.
3. Counting whole numbers from 1 to 6
4. Concept of addition

Directions:
1. Determine who should shake first. You can do this having each person roll the die, the high scorer goes first. Then rotate turns clockwise.
2. The first shaker rolls the die, looks at the surface facing upward, and then takes that many beans (markers) and distributes them one to each point (whole number) starting at 1. When she/he is through, she/he passes the die to the next player (to the left).
3. Players take turns shaking and filling their number lines until someone finishes filling all the numbers. At this age, the exact number to finish filling the line need not be shaken, but this rule can be added later i.e., if a player has 21 numbers filled on a 25-point line and shakes a 5, she/he is permitted to use 4 of the 5 to finish the fill. Later she/he would need to shake a 4 or less to be able to continue filling; she/he cannot use a number that is too large and must pass the die.
4. When someone finishes filling his or her number line, the game ends, although play can continue until everyone fills. But then the first filler has nothing to do. The first filler can be called a winner, "Chief Filler," or whatever designation you wish (if you feel one is necessary). The first filler can start next game.

Variations:
1. The length of the number line can be increased.
2. After a player shakes, she/he must orally announce how many "dots" she/he shook. Other players should correct him or her if she/he is wrong. The player could even lose a turn if she/he announces the wrong number.
3. A die with numbers instead of dots can be used.
4. The game can be designed for older students by using two different colored dice and designating one as positive and one as negative

"Race"

Equipment: One die, a car or other marker for each player, a number line with numbers 0 to 25 (or farther)

Players: Two to four works best. Grades 1 to ?

Skills: 1. One-to-one correspondence.
2. Counting whole numbers.
3. Adding whole numbers with one addend of size 1 to 6.

Directions: 1. All players put their cars (markers) at the start (zero) position.
2. Determine who shakes first.
3. The first player shakes the die and moves his or her car forward a number of spaces equal to the number of dots on the top face of the die. Then she/he passes the die to the player to the left who repeats process of shaking, moving, and passing the die.
4. Play continues until someone crosses the finish line. She/he is the "winner." However, play could continue to determine second place, third place, etc.

Variations: 1. The length of the race is variable. You can have a "Des Moines 100" or an "Omaha 500" if you wish.
2. For longer races, two or three dice can be used. Then players move forward the number equal to the sum of the dice.
3. Excitement can be added by having the teacher or students make special rules. For example, if one player must move his or her car to a space already occupied by another player, there is a "collision." The result of a collision can be as follows: One or both players lose a turn; the player simply cannot move his or her car to the occupied spot and must lose a turn; both players are "wiped out" and must leave the race; or any other variation you or the students decide or
4. After a player throws the die, she/he announces the number where she/he thinks she/he will end up–i.e., if she/he is on 18 and shakes a 4, she/he says, "18 plus (and) 4 is 22." If she/he is wrong, she/he must go back to 18 and lose this turn.

"Bunco" or "Twenty-One"

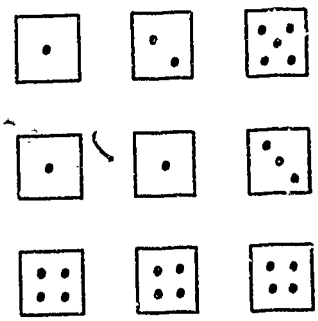
Equipment: Three dice, paper or a large supply of counters (corn, beans, chips, etc.) to keep score with.

Players: Any number, but two to four works best. Grades 1-4

Skills: 1. Counting by 1's, 2's and 5's up to 21.
2. Adding to get sums up to 21.

General Directions: The game is played in six rounds. Each round consists of the players' shaking for a designated number. In round one players start with 1's; each player adds the results of each shaking to his or her score until someone gets a total of twenty-one. Then round two begins in which people start shaking for 2's and continue until someone gets a total of twenty-one. Then round three, round four, round five, and round six follow.

Rules: 1. In round one the first person shakes the dice and counts the number of 1's showing. She/he gets one point for each 1 or five points for three of a kind other than 1's, and becomes an automatic winner of round one by shaking three 1's at once. This is called a "Bunco." (See Figure 2.)



is worth 1 point

is worth 2 points

is worth 5 unless you are shaking for fours. Then it is a "Bunco" worth 21 points.

Figure 2

2. If a player earns any points on the shake, she/he adds it to his or her total and *then gets to shake all three dice again.* The player continues to shake and add until no points are scored on a a shake, at which time she/he passes the dice to the next player.
3. Round one is complete when someone reaches or exceeds a score of 21 on his or her turn. Then everyone starts with a score of zero again and round two begins. Then you score *one* point for each 2 you shake. Three of a kind is still worth five except if you shake a "Bunco" of three 2's. Rounds three, four, five, and six follow the same rules with the corresponding number becoming the object worth points and determining a "Bunco."
4. Play is complete after six rounds, which produces six individual winners. The person who

wins the most rounds can be declared the overall champion.

"Boardwalk"

Equipment: Two dice, markers, and a game board for each player. The board will vary, depending on which arithmetic operations are used. See Figure 3.

2	3	4
5	6	7
8	9	10
11	12	+

Board 1

0	1	2	3
4	5	6	7
8	9	10	11
12	+	-	

Board 2

0	1	2	3
4	5	6	7
8	9	10	11
12	15	16	18
20	24	25	30
36	±	x	÷

Board 3

Figure 3

Board 1 is for addition only. Board 2 is for addition and subtraction, Board 3 is for all four operations.

Players: Any number, but two or three works best. The grade level depends on which board or skills are used.

Skills:
1. Adding, subtracting, multiplying, and dividing two numbers from 1 to 6.
2. Some decision making.
3. Intuitive probability.

General Directions: The purpose of the game is for each person to attempt to be the first to fill his or her board.

Rules for Board 1:

1. The first player shakes the two dice, orally adds the two numbers, and then places a marker on the sum on his or her board. Then the next player shakes and does likewise.
2. If a player shakes a sum that already has a marker on it, she/he cannot make a play and simply passes the dice to the next player.
3. The first person to cover all of his or her numbers wins. (You can continue to play for second, third, etc.)

Rules for Board 2:

Rules are the same as for Board 1 except that the player has a choice of adding or subtracting (smaller from larger) the two numbers she/he shook. However, the players should begin to realize that if they shake two 6's, then it would be better to cover the sum of 12 than the difference of zero on the board (if 12 isn't already covered) because there is only one way to get a sum of 12 and six ways to get a difference of zero.

You could permit a player to cover both the sum and the difference on each shake.

Rules for Board 3:

Rules are the same as for Boards 1 and 2 except that a player can add, subtract, multiply, and divide (disregarding those quotients that produce remainders or fractions) the two numbers she/he shakes.

Again, you can let the player cover only one number (a sum, difference, product, or quotient) on each turn, or you can let him or her cover all four answers if possible.

Variations:
1. If a player announces an incorrect sum (difference, product, or quotient) and another player catches the error, the player who made the mistake loses a turn or the player who catches the error gets to cover the number on his or her board instead.
2. Instead of playing against each other on separate boards, two players can work together on a single board against another team on another board.
3. One player can try to cover a board in as few shakes as possible, trying to establish a world record. It would be wise to have the student keep a record of all of his or her shakes to verify the results—i.e., first shake: 2, 6, second shake: 1, 4, etc.

MONEY, MONEY, MONEY!

by Kathy Reed

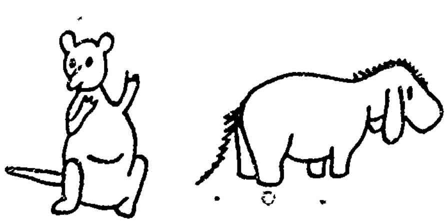
Money is a subject of interest to students and an important mathematics topic in the early grades. The author, a teacher at Sweetbriar Elementary School, Troutdale, Oregon, has put together a total theme unit to teach money skills.

Money! Everybody likes money! It is a favorite subject for youngsters, too. It "lends" itself to activities, centers, and games. A learning sequence for money, activities for identifying coins, counting in multiples, counting money, drill for money skills, applying money skills, and games involving money are presented by the author in this article.

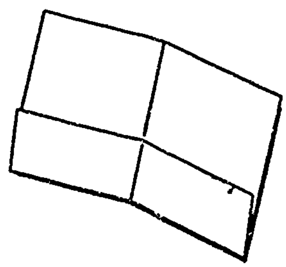
There are several things you can do to prepare your class and classroom for games and other activities (for any subject). The following section provides some helpful organizing hints.

PREPARING FOR ACTIVITIES

1. *Themes.* Students enjoy developing themes for units. For money, the theme created by the author's students is the Pooh characters. Things in the unit are grouped by skills (Pooh's Store, Eeyore's Restaurant, Kanga's drill, etc.).



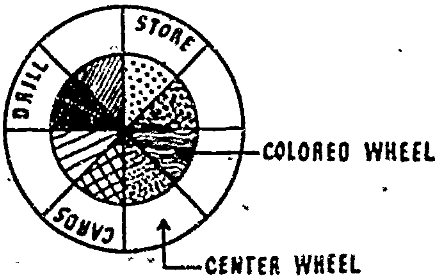
2. *Folders:* Each student has a folder for a unit. It can be made with 12" X 18" construction paper folded in half to be 12" X 9". A pocket is possible by using larger paper (fold the pocket up before folding the paper in half). The students keep their task card record papers and worksheets in the folder. A chart on the front allows the student to record what he/she did on a given day and the teacher to comment.



3. *Task Card Record Paper:* In the primary grades it is helpful to provide a record form for the students. This form will have the same theme symbol as the cards (Pooh, Eeyore, etc.). If there are varying levels of difficulty (A, B, C) for the cards, this level will be indicated. The record paper provides numbered blanks for the student to record his/her answers. Such a record form would help older students in recording their answers if they are not familiar with task cards and centers.

4. *Wheel.* Students need to know what activity they are to do on a given day. For the author, the "wheel" does this. For learning "centers" or "stations," students are grouped and each group has a color. The folders the students use are colored. The "wheel" has a large outer circle

and a small inner circle. The small circle is divided; each division represents a group and is colored accordingly. The large circle is divided; each division represents a "center" or activity and has the theme symbol for that activity. Each day the teacher rotates the inner wheel. In this way, students rotate through the centers, and they know what task they are to do each day.



5. *Groups:* The groups can be formed in several ways
   a. Four students: 1 "low," 1 "high," 2 "average"
   b. Three students: 1 of each "level," or all three about the same skill level.
   c. Two students.

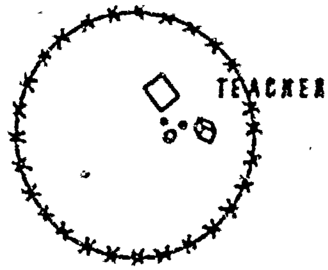
It helps to keep similar skill-level students together when the student's level will determine what tasks and/or activities and games he/she can do. Money is such a topic. *Warning:* It is a challenge to the teacher to find material easy enough for the "lowest" child and difficult enough for the "highest" child.

It is helpful to have low and high students together when the lower ones are apt to encounter reading or math skills beyond their present level while doing a larger, meaningful task

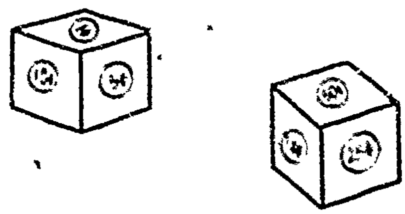
6. *Storage* It is helpful to keep the activities for a unit together in one place The students then know where the material is - and where to put it away It is easy for the teacher to check that everything is prepared and in place for the day's activities.

7. *Multiple Copies* If there is more than one copy of a game or drill activity, it helps to make each one a different color so that pieces and cards don't get mixed up

8. *Rules and Procedures* Students need to know the rules and procedures for the activities The teacher can introduce each activity in a class meeting but only introduce a few items in one session The rules or procedures should be explained, the activity should be demonstrated, and students should discuss potential problems ("How many can do this activity at once? What things are easily lost? How should it be put away? Is it designed for a particular skills level?")



9. *Behavior* The following are a few basic "ground rules" for games
   a. Anyone may play (you can't exclude someone unless the game's limit has been reached)
   b. Games with dice (unless foam) must be played on the carpet or a piece of felt

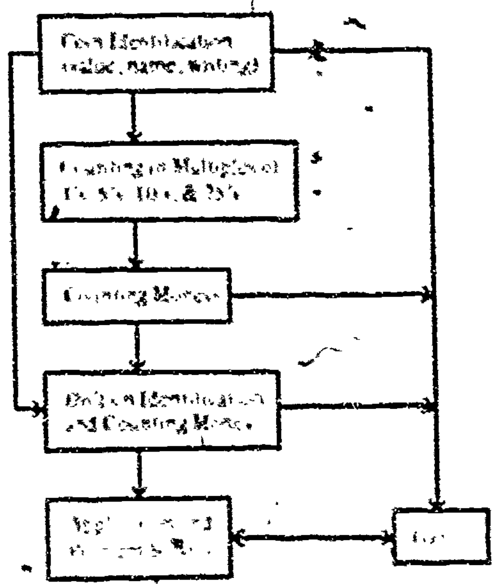


   c. The rules must be followed unless alterations were agreed to before the game was started
   d. All players help clean up
   e. Players must keep their voices low
   f. All players should be kind, thoughtful, and courteous

10. *Honesty* In the author's [illegible] has never been a problem even though [illegible] money is used in many of the activities. The [illegible] play a few "tempting" situations. The teacher always knows how much money is "out" and quickly checks as the activities come in. Occasionally there is a [illegible] for a dime or quarter however they're almost always found under a table, chair, or bookcase

A LEARNING SEQUENCE FOR MONEY

Money, like most [illegible], has a progression of skills. The author has found that when students have difficulty they often need to back off a step; the prerequisites have not been mastered. This flowchart is an example of a money sequence.



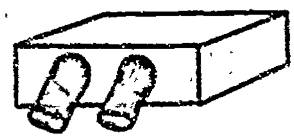
Coin Identification

Students need to know the name and value of each coin. They also need exposure to different … visual, tactile, … education, etc. The following activities are designed to build skill in coin identification.

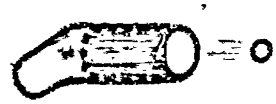
1. Matching. The student places the coin on a card with the name and value. Or he/she can place the … card beside the coin.

2. Hide Coin … Place the coins … on a table. The student … Have the student close his/her eyes while another child removes one coin. The student then looks and tries to determine which coin … The … students were …

3. The Feel-It Box. In this activity, a student places each hand through … holes covered with the tops of … socks. Two of each coin are in the box. The student … coin. To help in … a picture of the coins with the names and values should be available. The Feel-It Box is purely tactile and allows a student to use both hands to compare coins. It can be used for identifying … tactile … etc.



4. The Sock Box. The Sock Box is … covered with a ribbed sock. Two coins of each coin are placed in the Sock Box. A student uses one hand to select the correct coin. The Sock Box also can be used for … names, … writing, … etc.

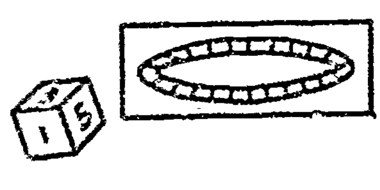


5. Behind the Back. This is very difficult … A student places his/her hands behind his/her back. The teacher places one coin in the student's hand. The student must then identify the coin.

When a student knows the size and weight of a coin so well that he/she can identify it by feel, he/she will probably make very few errors in identifying coins while counting money or making change. … should be … when a student can identify … coin's name and value while only feeling the coin. In … both the name and the value of the coin should be …

Multiple Counting

Before a student can … he/she must be able to count in multiples: 1¢s, 5¢s, 10¢s, 25¢s. Playing board games …



move the number of spaces you rolled), counting around the room, and finding missing numbers in a pattern (10, 20, ___, 40, 50, ___, ___, 80) are all good activities

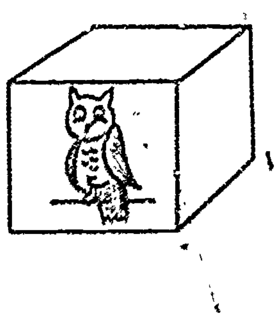
Another prerequisite to counting money is to "count on" in multiples (start at 50 and count by 5's; start at 75 and count by 10's) The student will also benefit from changing the multiples as he/she counts ("25, 50" *change to 10's* "60, 70, 80" *change to 5's* "85, 90, 95," etc.)

Counting Money

Counting money is fun and exciting for students. The author starts by having them count with one denomination ("We have lots of nickels. How much money do we have?") At first, most students simply count 5, 10, 15, 20, 25 ... s They often find they lose their place and must start over. Thus, they learn to make stacks of 10 pennies or 10 nickels. Dimes, quarters, and half-dollars are usually stacked in $1.00 piles by the students. It is usually best to start with pennies, nickels, *or* dimes. After each of these can be done alone, two of these varieties are mixed, finally all three denominations are used. After pennies, nickels, and dimes are mastered, the other coins are introduced, one at a time. The following are a few activities for counting money.

1 *Guessing Games* The students should learn ways to make $1.00, $2.00, and so on. The author's classes enjoy playing guessing games: "Ten of me make $1.00 Who am I?" "You have seven of me, this is more than $1.00, but less than $2.00. Who am I?"

2 *Owl's Money Box.* It is important that students use actual money (not play money) while learning to count money The author's class keeps money to be counted in Owl's Money Box. Owl is one of the Pooh characters, his picture is on the box. The amount in Owl's Money Box ... It starts with pennies, nickels, and/or dimes, it works up to all the coins and even dollar bills. The total amount will start under $1.00 and work up to several dollars. Students count Owl's money. When they have counted it, they show the teacher how they counted it, and he/she checks the result and discusses the process with the students



3 *Show an Amount* Students need practice in selecting coins to show a specific amount ("Show me 37¢") Students will progress from showing the amount with any coins they want to use to using the fewest coins, the most coins no quarters, and so on.

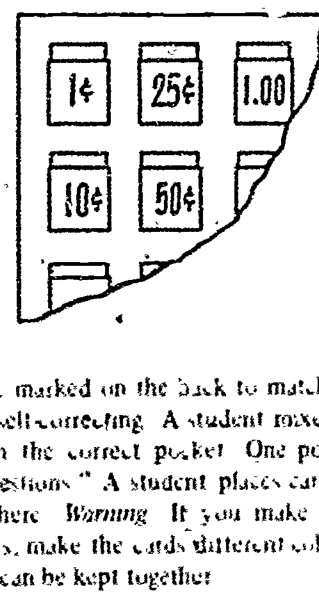
Solitaire Drill Activities

These are drill activities for coin identification, coin equivalence, and counting money that can be done alone.

1. *The Feel-It Box* (described under coin identification) can be used by students on their own.

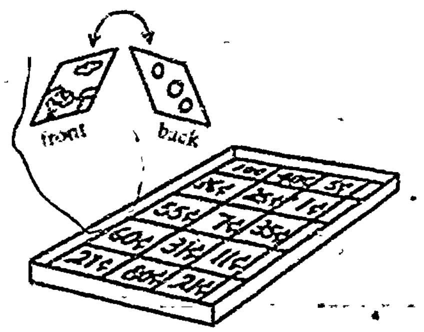
2. *The Sock Box* (described under coin identification) can also be done alone

3. *Kanga's Pocket Chart* This is an exercise for coin equivalence. Each pocket (library card pockets) is given an amount (1¢, 5¢, 10¢, 20¢, etc). Three by five-inch cards have the same amounts on them in several different forms ("dime", 10¢, $.10, a picture of a dime's tails side, another of a dime's heads side, another of two nickels, and so on).



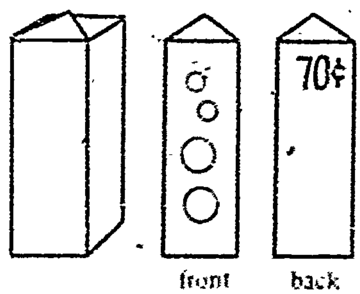
The cards are marked on the back to match a code on the pockets for self-correcting A student mixes the cards and puts them in the correct pocket One pocket is marked "Kanga's Questions " A student places cards he/she is unsure about here *Warning* If you make several Kanga's Pocket Charts, make the cards different colors so that each chart's cards can be kept together

4. *Picture Puzzle in a Box* This is a drill for coin equivalence and/or counting money Students work this puzzle by matching the coins on the picture square to the totals in the box To make a puzzle, cut two pieces of railroad board to fit exactly in a box (a hosiery box works well) Divide the two pieces of railroad board into squares about 5-8 cm square Glue one in the bottom of the box On the other piece, glue a picture the students will like Cut the picture into the squares you marked On the back of each picture square, stamp the values, write the amount



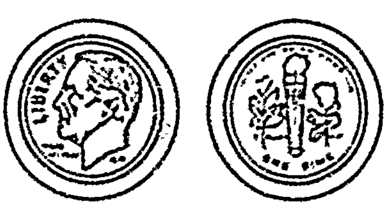
on the corresponding square in the box—be careful to keep the picture in the correct order.

5. *Match Box Drill:* This is an exercise for counting money. Take a decorative match box and cut railroad board strips to fit in the box. On one side money is pictured (use the money stamps). On the back of the card, write the correct total for the coins pictured on the front. The student must count the money and check his or her answer against the answer on the back. You can use different colors of railroad board for the cards to show levels of difficulty; this will make the drill suit more students.



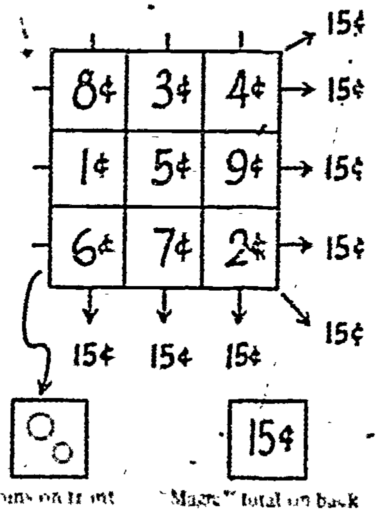
Other Drill Activities

1. *Heads-Tails Match* (coin identification). Students match discs with the heads and tails (name or value can also be used) of the coins on them. This can be self-correcting by coding the backs of the discs.



2. *Owl's Money Box* (described under counting money). This can be self-correcting if the students have access to a card with the correct total.

3. *Magic Squares* (counting money and problem solving). A 3 X 3 array is the easiest magic square to solve. To make one, cut nine small squares and stamp the values needed for a magic square on each square (1¢ through 9¢ will total 15¢, 4¢-12¢ will total 24¢, 29¢-37¢ will total 99¢, etc.). Since you can make several different value totals (15¢, 24¢, 99¢, etc.), it will help to mark the back of each square with the total to be made. In this way, a student can be sure that his/her pieces are correct for the magic square he/she is attempting, and any misplaced squares can be returned to the correct puzzle. To correctly work a magic square, the students should arrange the squares into a 3 X 3 array so that the total is the same in all directions (rows, columns, and diagonals).



Applications

There are endless ways to apply money to "real" situations. Most are fun, relatively easy to set up, and inexpensive. The author's students have two favorite applications which are explained in this section. Other "real" situations are suggested.

1. *Pooh's Store:* Pooh owns the store in the author's classroom. It's a grocery store. To make one, first collect grocery items (open cans and boxes from the bottom to preserve the fresh appearance and the price). Check that each item for your store has a price clearly marked on it and that identical items have the same price. These grocery goods should be displayed in "departments." Cardboard boxes can be stacked, a bookcase can be used, or a special "store" can be built. Once you have a store, write task cards that are at an appropriate level for your students. (*Warning:* Make them much easier than you think is necessary; overshooting is a common problem.) Sample cards

GO TO POOH'S STORE B-1

A. How much does a can of applesauce cost? ____________

B. How many dimes must you give Pooh to buy the can of applesauce? ____________

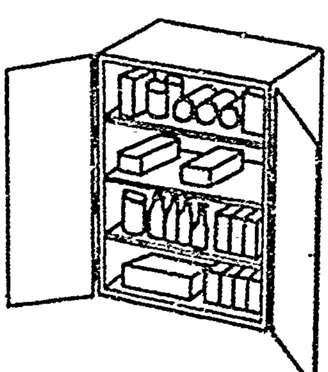
GO TO POOH'S STORE B-9

A. What is the most expensive item in the spice department? ____________

B. How much does this item cost? ____________

C. You have $2.00. How much change should you receive if you buy this most expensive item in the spice department? ____________

At the store, students learn to read prices and labels (brand product, size, servings). They learn to categorize grocery products into departments. They work on counting money ("Do you have enough money to buy eggs? How much extra do you have? How much more money do you need?"). They work with coins ("What coins could you give Pooh to pay for the jello? What coins should Pooh give you as your change?"). Students can take inventory, work on wholesale and retail comparisons, and study marking, overhead costs, or discounts for bulk buying. They can compare brand names and sizes for the best buy or study the metric system with grocery items.



2. *Eeyore's Restaurant.* Eeyore's Restaurant is a favorite with the author's students. The restaurant provides great practice in reading a menu, categorizing foods (breakfast, beverage, dinner, etc.), identifying coins, totaling amounts, counting money, making change, and even tipping and manners. All you need for a restaurant are a few menus from a local restaurant, some play money, and some task cards. Select a restaurant whose menu allows the prices and menu skills you want for your students. Remember to keep the task cards simple. Sample task cards:

EEYORE'S RESTAURANT A-3

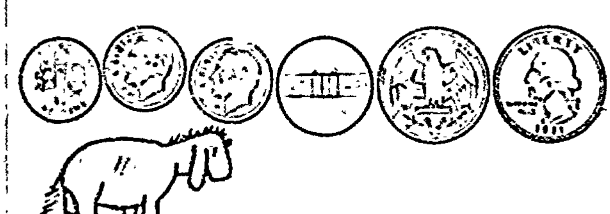
What does *Beverages* mean? ____________

____

EEYORE'S RESTAURANT B-5

Pooh wants pancakes (with honey) and a small glass of milk. What would his total bill be? ____________

He has



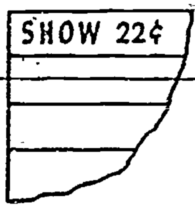
Does Pooh have enough money for his breakfast? ____________

A field trip to a restaurant is a good conclusion to the money unit.

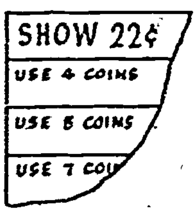
Similar applications are possible. How about a hot dog vendor store, an ice cream shop, or a toy store? Or you can get into the actual business with a student store, popcorn sale, or other fund-raising projects.

Problem Solving

1. *Show* ______¢. Students use the coin stamps to show different ways to make a given amount (such as 45¢). A more advanced skill is to determine how many different ways there are to make a value.



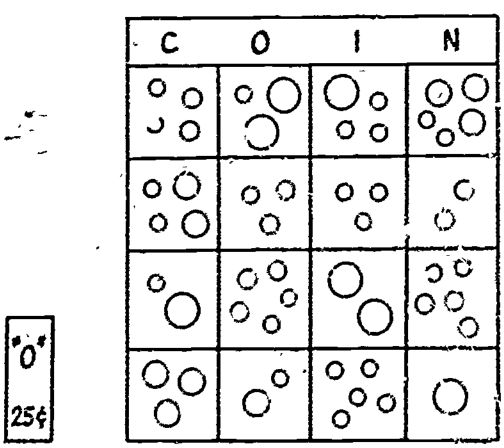
2. *Use* ______ *Coins To Make* ______¢: Students again show ways to make a given value (such as 22¢) but using a specified number of coins (such as 4 coins–2 dimes and 2 pennies; 5 coins - 1 dime and 2 nickels and 2 pennies, or 7 coins - cannot be done). In making such a worksheet, be sure to include ones that can't be done.



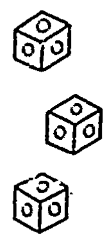
Games For Small Groups

Some of the following games use *coin dice*. Coin dice are six-sided foam dice with pictures of each coin (penny, nickel, dime, quarter, and half-dollar) and duplicates of one of them. The dice can be covered with clear Con-Tact paper to protect and secure the pictures.

1. *Coin Bingo* (counting money): For Coin Bingo, each card is divided into 16 squares. Coin stamps are used to show values which are also written on cards (25¢, etc.) to be used in "calling." The game can be simplified by making columns. "C" (1¢-19¢), "0" (20¢-39¢), "I" (40¢-69¢), "N" (70¢-$1.00). The call cards are then marked with the clue letters. The game is played like other bingo games.

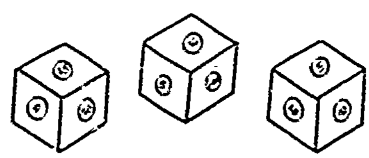


2. *Value Bingo* (counting money). For Value Bingo, each card is divided into 25 squares. Each square has a value that can be made by a combination of three coin dice. The call is made by rolling the dice.



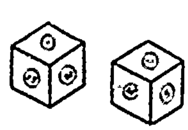
3¢	16¢	27¢	40¢	51¢
25¢	36¢	7¢	70¢	21¢
12¢	35¢	FREE	45¢	31¢
30¢	65¢	20¢	60¢	61¢
52¢	11¢	55¢	75¢	15¢

3. *Dice Game:* The object of this game is to earn the most points by identifying the total value of a roll of three coin dice. A player earns one point if he/she correctly gives the total of his/her roll. An opponent earns two points if he/she discovers a player's error and gives the correct value. Play rotates. The first player to reach 12 points, or the person with the most points when time is called, is the winner.



4. *Tic-Tac-Toe* This game is played with two coin dice and one tic-tac-toe board (with amounts that can be

2¢ 55¢	30¢ 50¢	10¢ 35¢
20¢ 35¢	51¢ 75¢	6¢ 60¢
15¢ 75¢	11¢ 20¢	26¢ $1.00



made with the two dice written in the squares). The game is faster if two amounts are given on each square, however, once the square is covered, it can't be used again. Each player needs markers (different colors). A player rolls the two dice and gives the total. If he/she is correct, he/she covers the value on the board. If the value isn't on the board, if the value has already been covered, or if the player declares the wrong total, the player loses that turn. Play alternates until someone makes a tic-tac-toe or until a "cat's" game is called.

5. *Cards:* Use the coin stamps. Make a series of books—four cards with the same value (in different forms). You will need 10-13 books to make a deck. Now, with this deck you can play several games. Here are a few suggestions:

a. *"Authors" Style:* Deal six cards to each player. Player A asks for a value he/she has ("Do you have 50¢?"). Other players must give Player A the value he/she asked for. If Player A receives the value he/she wanted, he/she asks again. If no one has the value, the player draws a card and this ends the turn. The object is to make books of four cards with the same value. The winner is the person with the most books when one player is out of cards.

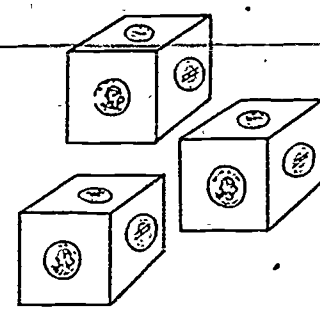
b. *"War" Style:* Deal all the cards. Each player places his/her cards in a stack. Each player turns over a card. The person with the highest value takes all the "up" cards. In case of a tie for highest value, there is a war. For this, the players involved place two cards face down and then turn one card face up. The person with the highest value on the new "up" card wins the round. Play continues until someone is out of cards. The winner is the player with the most cards.

c. *Rummy:* Deal seven cards. This game is played like Rummy. Books are made with three or more cards of the same value, or three or more cards that total either 75¢ or $1.00. Players alternate turns. They may pick up the top card from either the discard pile or the draw pile. A player ends his/her turn by discarding one card. The winner is the first person to use all his/her cards in books.

Games for Large Groups

1. *Bingo.* Either Coin or Value Bingo (described in small group games) may be played by a large group. You'll need enough bingo boards so that everyone has a board and markers to cover the called values.

2. *Table Dice Game* (counting money). For this game, each team (table of 3-4 students) needs three coin dice. At the same time, each team rolls their dice and determines the total value. Rotating around the room, one person from each team gives his/her team's roll and total ("We rolled one quarter, one half-dollar, and one dime: the total is 85¢."). After each team has declared its totals, two points are awarded to the team with the highest total and two points are awarded to the team with the lowest total



If there is a tie, each team involved earns the points (they don't play it off or share the points). Play continues as long as time permits. Time can be saved by having each team write and display their total on an individual chalkboard (this eliminates the declaring time).

3. *How Many Ways Can We Make ______¢* (problem solving and counting money). For this activity it helps to have coins and/or overhead coin pictures. The object is to determine all the coin combinations to make a given value (such as 45¢). Students take turns suggesting and/or showing a way to make the given value. Encourage students to look for patterns or strategies (such as starting with all pennies and working up). It can be a game with each team giving combinations. A correct combination is 1 point, a repeat is 0, and an incorrect combination is -1 point. A team earns 2 points if they correctly say that there are no more combinations.

4. *20 Questions* (problem solving and counting money). This activity is a favorite one in the author's classroom. You will need a supply of coins. The object is to determine the exact coins and the total value of the coins (1-10 coins) that the leader is holding. The leader secretly selects 1 to 10 coins. He/she announces how many coins he/she is holding. The players ask questions that the leader can answer with yes or no ("Do you have two or more pennies? Do any of your coins have a value greater than 25¢? Do you have either 0 or 1 nickels?"). Questioning continues until someone can guess the total value. That person becomes the new leader.

The 20 Questions activity can be simplified by writing the coins on the board as they are determined (write "two pennies" when a questioner has determined this information). It can also be simplified by showing the coins as they are determined. When it is found that there are two pennies, the leader places the two pennies on the floor, a table, or an overhead. 20 Questions becomes more challenging by keeping a tally of the questions. If the value isn't determined within 20 questions, the leader selects new coins and the game starts over.

CONCLUSION

Money is an enjoyable unit for students. It is a good topic for introducing your class to activities and games. You can make these for a minimum financial investment and use readily available materials. It takes time to develop the material, but if you laminate it or protect it with clear Con Tact paper, your games and task cards will last several years. Coin stamps (there are a heads set and a tails set) are available through various commercial outlets. These stamps are worth their cost; you'll find yourself using them over and over again. Your students will also enjoy using the coin stamps.

As you develop and/or expand your money unit, you'll find one activity inspires another. Good luck and have fun!

SOME METRIC GAMES AND ACTIVITIES

by Nancy Eure and Curtiss Wall

These metric games are great for elementary and junior high students. They have been tested and refined in a fourth grade classroom. Nancy Eure is with the Newport News Public Schools, Curtiss Wall with Old Dominion University.

"Make a Meter" (converts within the metric system)

Materials: A gameboard for each player consisting of a 4 X 5 array of squares (Figure 1), a set of cards (in a size that will fit in the squares on the 4 X 5 array) for each player with the following written on them.

200 mm	300 mm	80 cm	1 dm
900 mm	30 cm	7 dm	5 dm
50 cm	700 mm	4 dm	600 mm
20 cm	8 dm	100 mm	90 cm
6 dm	40 cm	500 mm	5 dm

Procedure: Each student takes a set of cards and arranges them face up on his or her board. Players then exchange boards. Each player in turn moves one card horizontally, vertically, or diagonally one space so that cards in adjoining squares total one meter. The player then removes the cards totaling one meter from the gameboard. Each player makes only one move per turn, regardless of whether he or she can remove cards from the board. Cards cannot be placed on occupied squares. The first player to remove all of his or her cards from the gameboard is the winner.

In the event that, on the first turn, a player cannot make a meter, he or she may interchange any two cards on the board as his or her turn.

This game may be adapted to mass or capacity by changing the unit of measurement to gram or liter.

A similar activity may be found in *Mathematics Laboratories, 150 Activities and Games for Elementary Schools*, ERIC Information Analysis Center for Science, Mathematics, and Environmental Education, p. 41.

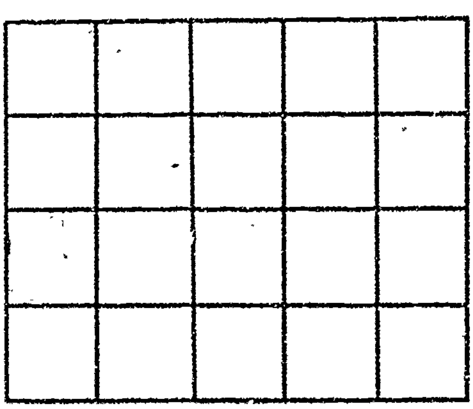


Figure 1

"Metric Barnyard" (measures to the nearest centimeter)

Materials: One die labeled 1-6, a different colored chip for each player, cards as described below, a gameboard as pictured in Figure 2, paper fasteners, strips of tagboard (5 cm, 10 cm, 15 cm, 20 cm, 25 cm, and 30 cm long with a hole punched at each end) as described below.

Lumber Cards (seven of each): 5 cm, 10 cm, 15 cm, 20 cm, 25 cm, 30 cm

Lumber
- Eight 30-cm "boards"
- Twelve 25-cm "boards"
- Sixteen 20-cm "boards"
- Sixteen 15-cm "boards"
- Eighteen 10-cm "boards"
- Sixteen 5-cm "boards"

Chance Cards
- (Two of each)
  - You get a loan from the bank. Pick an extra lumber card.
  - You get a tax refund. Pick one lumber card.

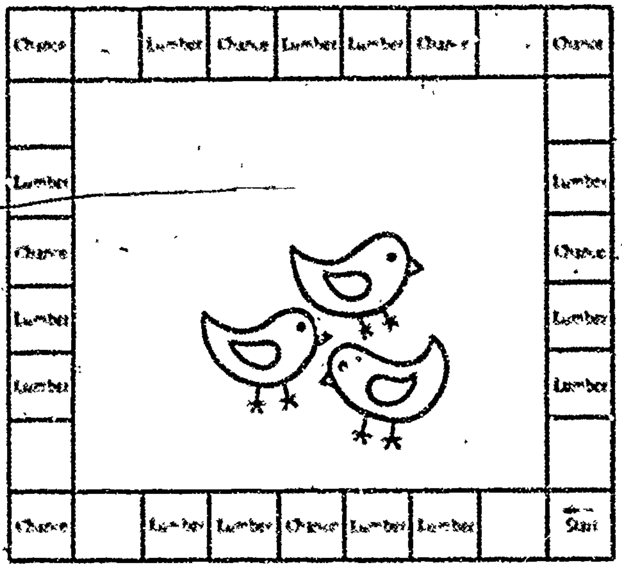


Figure 2

Your neighbors come over to help you. Pick a lumber card.

You inherit money from a rich uncle. Pick two lumber cards.

Your son and daughter are home from school. They want to help. Pick two lumber cards.

You get a new saw. Pick two lumber cards.

There is a sale at the hardware store. Pick two lumber cards.

The lumberyard is having a sale. Pick two lumber cards.

[illegible]

Cattle wreck your fence. Remove the last piece of lumber.

There is a crack in your last board! Remove it.

You need a rest. Lose one turn.

Time to pay your taxes. Lose one turn.

It is raining. Lose one turn.

It is time to eat dinner. Stop work.

Procedure Each player selects a chip and places it at *Start*. Players in turn roll the die and move the correct number of spaces. If a player lands on a space marked *Lumber*, he or she picks a *lumber* card and takes a "board" of the appropriate size. He or she fastens the "boards" together with the [illegible] and tries to build a "fence" around the barnyard animals. If a player lands on a space marked *Chance*, he or she picks a *chance card* and does what the directions say. The first player to complete a fence that surrounds the barnyard animals is the winner.

"Race the Rabbit" (measures to the nearest centimeter)

Materials Tagboard, 48 cards, centimeter ruler, tokens for each player, sheet of acetate large enough to cover tagboard, grease pencil

Procedure Draw a race course using various line segment lengths as pictured in Figure 3. Make cards as indicated.

- 4 Lose turn
- 4 Free turn
- 10 Go back one space (to the previous corner)
- 30 Go forward X centimeters

Students take turns drawing cards and moving as directed. The first to reach the finish is the winner.

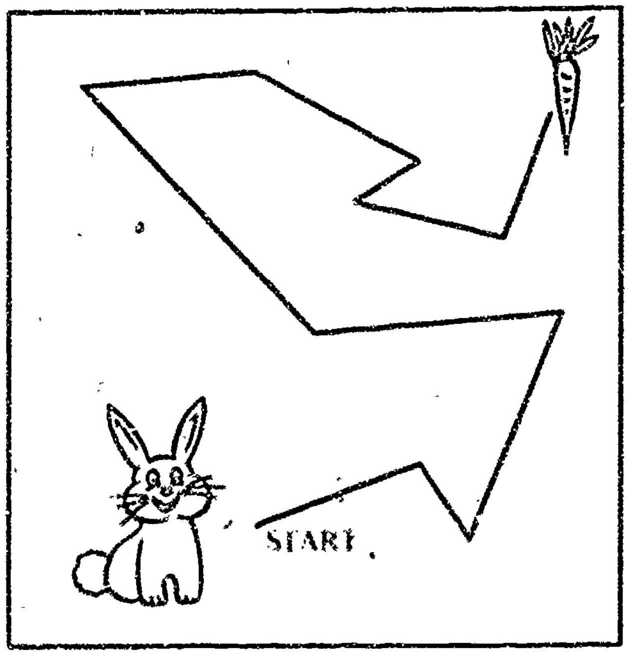


Figure 3

"Ellie English"

Objective To know correctly [illegible] use of the metric system

Procedure Ellie English does not want to change over to the metric system. Don't get stuck with Ellie!

This game is played like "Old Maid."

Materials [illegible] cards, [illegible] of the following:

A quart of milk	955 ml
1 liter	Water [illegible]

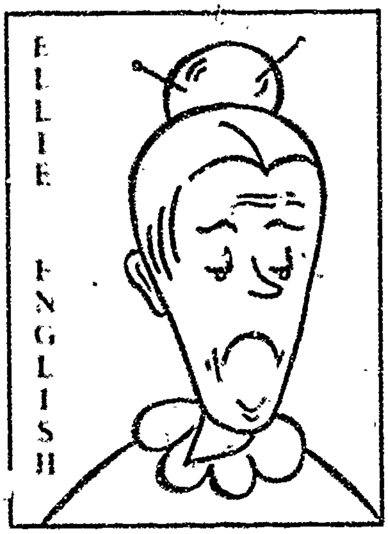


Figure 4

1 teaspoon	0°C
5 ml	Mass (weight) of a stick of butter
Temperature of boiling water	100 grams
100°C	Thickness of a slice of bread
Normal body temperature	1 cm
37°C	Height of a door
Speed limit on a turnpike	2 meters
75 km/h	Room temperature
Mass (weight) of U.S. dollar bill	20°C
1 gram	Thickness of a dime
Speed limit in a school zone	1 mm
40 km/h	Weight of Miss America
Bottle of 7-Up (regular size)	50 kg

Note. This game may be played as a concentration game by removing the Ellie English card.

"Metric Bingo" (uses appropriate units of measure)

Materials. One gameboard as pictured in Figure 5 for each player, clips, cards with the following written on them

Rice	Syrup	Can of soup
Salt	Soap	Height of door
Tree	Pills	Rock
Peanut butter	Box of jello	Can of peas
Oil	Bottle of vinegar	Butter
Window	Can of dog food	Thread
Telephone pole	Length of car	[illegible]
Milk [illegible]	Sack of flour	Can of coffee
Gas tank	Height of room	Can of motor oil
Lake	Can of corn	Cup

meter	gram	liter	gram
gram	liter	meter	meter
meter	gram	liter	meter
liter	meter	gram	liter

Figure 5

Package of sugar	Length of room	Girl's weight
Liquid medicine	Width of room	Steak
Liquid detergent		

Procedure. Each player is given a gameboard. Shuffle the cards and lay them face down. Each student in turn draws the top card and covers the appropriate unit of measure on his or her board with a clip. The first student to get four in a row, column, or diagonal is the winner.

Another approach to this skill may be found in *Fun and Games with Metrics,* Prentice-Hall Learning Systems, Inc., p. 55.

"Beware" (measures to the nearest centimeter)

Materials. A gameboard as pictured in Figure 6, a marker for each player, cards with pictures or figures to be measured.

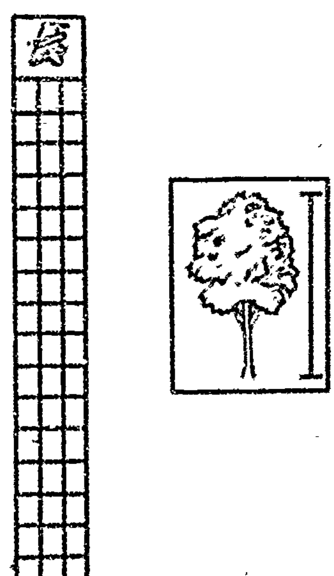


Figure 6

Procedure. The teacher should begin by making a set of cards from magazine pictures or pictures from old work books. To clarify which dimension is to be estimated, the teacher should mark a line on the card indicating the distance to be estimated. Each player puts his or her marker at the bottom of the gameboard. Players in turn pick a card and estimate the length of the figure in centimeters. The number of centimeters they are away from the actual length of the figure is the number of spaces that they must advance toward the witch. The winner is the student who manages to stay away from the witch for the longest time.

CONCENTRATION IN THE CLASSROOM

by Robert McGinty, Jane Swafford, and John Van Beynen

The game "Concentration" is familiar to most students. Little class time needs to be spent on the rules of the game, and, therefore, more can be spent on drill or reinforcement. This article suggests an inexpensive, yet effective way to make "Concentration" a whole class activity. The authors are all affiliated with Northern Michigan University.

Teachers are always on the lookout for motivational activities to supplement their normal classroom routine. In addition to the motivational aspect, it is more important that the activity provide the students with some type of instruction—i.e., concept formation, drill, reinforcement, remediation, etc. One such activity that fits these requirements is the game of "Concentration." Most students are familiar with this game, so the amount of explanation needed is minimal. The way students normally play the game is to use an ordinary deck of playing cards with the cards spread out face down in front of them. The number of players can vary from 2 to 6. The first person chooses two cards and turns them face up; if the two cards match (both Kings, both 7's, etc.), then the person keeps that pair of cards. If the cards do not match, then the cards are returned face down to their original position. The second person then chooses two cards and turns them face up, and the play proceeds until all of the cards have been matched. The player with the most pairs of cards is the winner.

You can alter the game of "Concentration" to fit your needs. First, make a set of cards from tagboard or a similar material. If you want your students to play at their desks, make the cards about the size of ordinary playing cards. If you want the game to be a whole-class activity (one side of the room against the other), use two sets of cards about 10 cm X 15 cm (4" X ndex cards work well). Prepare one set of cards as the p ig deck. Number the second set of cards 1 through n, where n is the number of cards in the playing deck. Shuffle the playing deck and then clip a playing card upside down to the back of each numbered card. Tape the cards to the chalkboard as shown in Figure 1. If the cards are taped as shown, then all you need to do is raise the cards up as the numbers are called out. Remember to clip the playing card upside down so that when the number card is raised, the hidden playing card will be seen in an upright position. When two cards match, they can be removed from the board and put in the pile of the appropriate team.

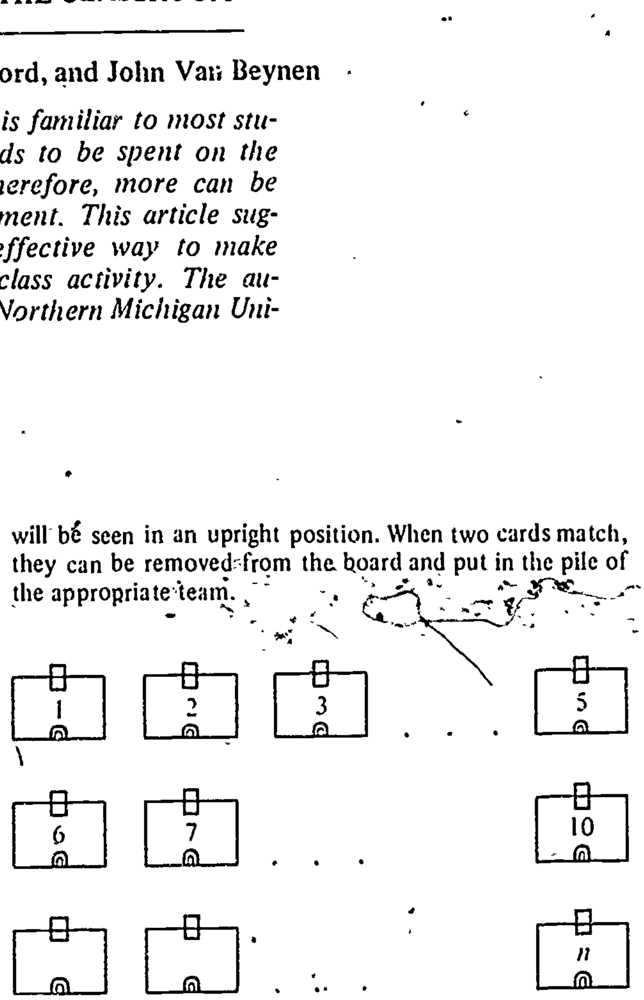


Figure 1

The number of cards needed will vary, depending on what version of "Concentration" you use, usually between 12 and 36 cards is sufficient. Once the numbered cards have been made, you can invent many different games for your classroom. The following include some of the many variations we have seen:

1. To make an addition game, on one card write two addends such as 2 + 3, and on the match card write the sum 5. For multiplication, on one card write two factors such as 2 X 3, and on the match card the product 6. Similarly, cards can be made for subtraction and division facts.
2. For number recognition, on one card write the numeral 6, and on the match card use 6 dots or the word *six*.

3. On one card write the name of a geometric figure, and on the match card draw the geometric figure.
4. On one card write a fraction, and on the match card write an equivalent fraction, an equivalent decimal, or an equivalent percentage, or draw a picture of the original fraction.
5. On one card write a missing addend sentence such as $3 + \square = 8$, and on the match card write the missing addend. Or write a missing factor sentence such as $\square \times 7 = 56$, and on the match card write the missing factor.
6. On one card write an indicate square root such as 49, and on the match card the root 7, or use a perfect square like 25, and on the match card 5^2.
7. On one card write one half of an even number such as ½ X 42, ½ X 80, ½ X 16, etc., and on the match card write the answer. Odd numbers can also be used.
8. On one card write a metric unit such as 1,000 metres, and on the match card write an equivalent like 1 kilometre, or use English units and their equivalents.

GAMES AND ACTIVITIES WITH CARDS

by Donald L. Zalewski

This series of card games, designed for junior and senior high level students, uses regular playing cards. Variations are provided for each game to keep even the strongest math students challenged.

The first three "War" games can be played with an ordinary deck of playing cards, each card taking on these values: Ace = 1, Deuce = 2, Trey = 3 . . . J = 11, Q = 12, K = 13. The object of each game is to capture the other players' cards by playing a card or a combination of cards that has a larger value than those of the opponents.

All three games proceed best with two to four players although more can play.

Galaxy I—"Number War" or "Numero"

Skills Comparing values [illegible]

Directions 1. Deal the whole deck out to the players. Each player makes a pile face down without looking at them.

2. Each player turns the top card of his/her pile face up.

3. The player who has the highest value captures all the face-up cards and adds them to the bottom of his or her pile. Then each player turns up another card and the procedure is repeated.

4. In case of a tie for high, each person involved in the tie puts one card face down and another card face up. The person with the highest face-up value captures all the cards that have been played.

5. The game is over when one of these conditions is reached:

a. One player, the "Star" or "Space Ace," has captured all the other players' cards.

b. One player runs out of cards. The "Star" is the person with the most cards.

c. A time limit is reached. The "Star" is the same as in 5b.

Variation 1. Let red cards assume negative values, i.e., the four of diamonds or hearts has a value of −4, but would capture any red 5 to [illegible] or King.

Galaxy II—"Total War" or "Suma"

Skills Finding sums for whole numbers [illegible] Mental [illegible]

Directions 1. Deal out the whole deck to the players. Each makes a pile [illegible] down without looking at the cards.

2. Each player turns two cards face up and announces the sum of his/her two cards [illegible] The player with the highest sum captures all [illegible] face-up cards [illegible] them to the bottom of his or her pile.

3. In case of a tie for the highest sum, the tied players each turn up two more cards and the highest sum of the two new cards takes all the cards, including the cards used to break the tie.

4. If the player with the highest sum [illegible] captures the face-up card.

5. Play continues until a "Star" is determined [illegible]

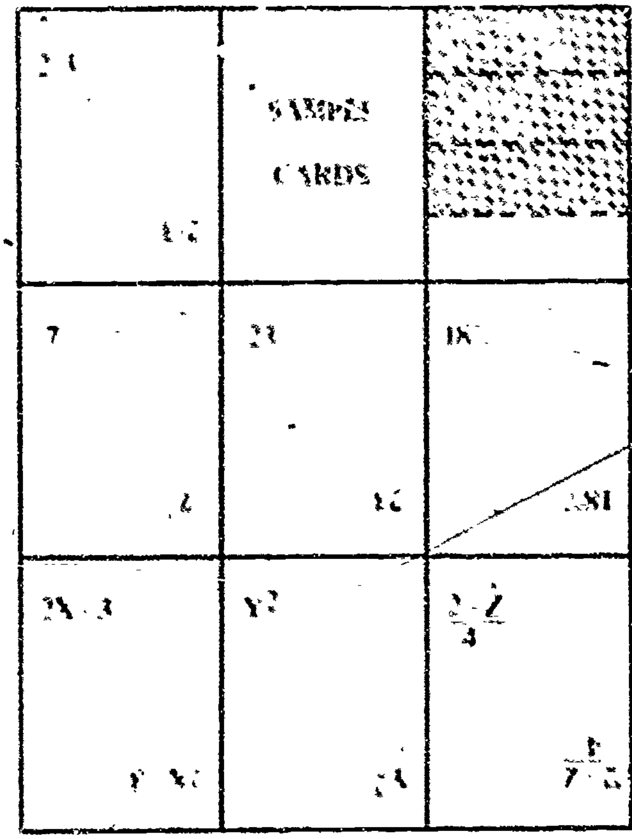
Variations. [illegible]

1. Change the game to "Ditto" by finding the difference between the larger card and the smaller card that were turned up. The player with the greatest difference captures the other cards.
2. Change the game to "Multo" by finding the product of the two cards turned up. The player with the greatest product captures the other cards.
3. Change the game to "Divvo" by finding the quotient of the larger card and the smaller card. The player with the largest quotient captures the other cards. If you disregard remainders, the quotient of a 7 and a 4 is 1, as is the quotient of a Queen (12) and a 10. If you want to also consider the remainders, then 7 ÷ 4 = 1 r 3 would beat Q(12) ÷ 10 = 1 r 2, but would not capture 5 ÷ 2 = 2 r 1.
4. Change the game to "Fracto" by finding the largest fraction that can be made with the two cards (very similar to "Divvo"). The player with the largest fraction captures the other cards. The [illegible] 7 and a 4 would produce 7/4, which would capture 8/5 but would not capture 9/4 or [illegible]. The largest possible fraction [illegible] King Ace combination [illegible] [illegible] would occur when a pair of [illegible] turned up [illegible].
5. [illegible] could play [illegible] by letting the person with the smallest [illegible] capture the other card.
6. [illegible] from the Jack and a 7 would produce 3 in "Ditto", [illegible] "Multo", [illegible] "Divvo" and [illegible] "Fracto".
7. To encourage speed in calculation [illegible] for the highest value, the first of the [illegible] players who [illegible] announces the sum (difference, product, or quotient) captures the cards.

Galaxy III Algebra

Skill: Evaluating algebraic expressions

Extra Materials: In addition to a regular deck of playing cards, a deck of cards with algebraic expressions on them [illegible].



Directions:

1. The entire deck of regular cards is dealt out to the players, who then make piles face down without looking at them.
2. The algebra cards are shuffled and put face down on a single pile.
3. Each player turns one of his or her cards face up. Then one algebra card is turned face up.
4. Each player evaluates the algebraic expression by substituting his/her card value in for the unknown. For example, if a player has a 5 up and the algebra card is "2x - 3," the person has a score of 7. The player with the highest score wins the face-up cards of the other players. The algebra card is put on the bottom of the algebra card pile.
5. In case of ties for the highest score, each person who is tied puts a second card face up and evaluates the algebraic expression with the new card's value. The highest score takes all the face-up cards.
6. Play continues until a "Star" is determined (see Galaxy I).

Variations: 1. Use the red cards to represent negative values. Then a red 5 and an algebra card "$2x - 3$" would produce a score of $2(-5) - 3$ or -13. But "$3 - 2x$" would produce a score of $3 - 2(-5)$ or 13.

2. In case of ties for high, the *first* person to announce his or her score correctly gets the face-up cards.
3. If the person with the high score announces his or her score incorrectly, the first person to correct the error gets to take the face-up cards.

Extra Note—Variations on Materials: Some of the "War" games can develop higher level skills of working with fractions and decimals by making separate decks of fraction cards and decimal cards. You could even make percentage cards. (See sample cards.) Each deck can be used separately as the games are played, or you can mix any combination of the decks. Thus, in "Numero," players could end up comparing 3/4 to 30% to 0.7 to decide which person had the highest value.

"Equals"

For two to four players

Skills: Combining whole numbers through the basic operations

Goal: To combine the value of your cards by adding, subtracting, multiplying, dividing, or any combination thereof to produce a given value.

Materials: A regular deck of cards (with the face cards removed if you wish to keep values small)

Directions: 1. Each player is dealt four cards and then one card from the deck is put face up in the middle.

2. Each player attempts to find a combination of *all* of his or her four cards to match the value in the middle. For example, if a player has 2, 3, 7, and 9, and the card turned up in the middle is a 10, the player could get 10 by $9 + 7 - (2 \times 3)$. If a 5 came up in the middle, the player could make it with $(9 + 7) \div 2 - 3$.
3. The first person to match the face-up card's value must explain how he/she got the value. If correct, the person gets to take his or her four cards and the face-up card and put them in a "winnings" pile. Then the player takes four new cards from the deck and turns up a new card in the middle. Play continues as before.
4. You may challenge another player's combination. If you are correct, you get the other player's four cards for your "winnings" pile. If you are wrong, you must give him or her your four cards for his or her "winnings" pile.
5. If no one can make a combination to equal the center card, a new center card is turned up.
6. The game ends when there are not enough cards left in the deck to give a player a new hand and a face-up card (in other words, there have to be at least five cards). The winner is the player with the most cards in his or her "winnings" pile.

Variation: 1. You might have each player use only three cards, or you could use more than four cards. But six or seven cards makes the game too dragged out and discourages using multiplication and division. (Calculators would help though.)

"Duel--50"

For two players.

Skills: Mental addition of integers. Strategy

Goal: To make a sum as close to (or equal to) 50 as possible so that your opponent cannot play without making the sum more than 50.

Materials: A deck of playing cards. (You can remove face cards if you wish, otherwise, J = 11, Q = 12, K = 13.)

Directions. 1. The dealer gives each player three cards.

2. The nondealing player starts by laying down a card face up, announcing its value, and taking a new card from the deck. The dealer plays a card, announces the sum of the two cards, and draws a new card. Then the nondealer lays down another card, announces the sum of the three cards, and draws a new card. Play continues until one player announces a sum less than or equal to 50 and

the other player cannot add on another card without making the sum exceed 50. The last person to make a play takes all the face-up cards and puts them in his or her "winnings" pile. The other person starts a new round.

3. The game ends when the players run out of cards. The last person to play a card gets the last face-up pile.
4. The winner is the player with the most cards in his or her "winnings" pile.

Variations: 1. More strategy can be applied if more cards are dealt to each player.
2. Let the red cards have negative values.
3. Play up to 75 or 100.

Sample Play: Opponent plays 10; announces "ten." Dealer plays 9; announces "nineteen." Opponent plays J (J = 11); announces "thirty." Dealer plays 7; announces "forty-seven." Opponent plays 6; announces "forty-three." Dealer plays 4; announces "forty-seven." Opponent plays 2; announces "forty-nine." The dealer only has 2, 8, and 8 left in his or her hand, so she/he can't play. The opponent takes the cards and puts them in his or her "winnings" pile. The dealer starts a new round by putting out either the 2 or one of the 8's. (Obviously, it would be better to keep the 2 for later.)

"Matchsum"

For two to five players

Skills: Mental addition of integers. Strategy.

Goal: Use a card in your hand to equal the sum of two or more cards on the board.

Materials: A deck of playing cards. Let the red cards have negative values.

Directions: 1. The dealer gives each player four cards and also puts four cards face up on the table.
2. The first player tries to find *two cards on the table* whose sum is equal to *one card in his or her hand*. If he/she can make a match, he/she lays down the card, states the sum, and puts the three cards involved on a "winnings" pile.
3. If a player cannot make a match, he/she discards one card by placing it face up with the other cards on the table.
4. Obviously, there will not always be four cards on the table. If no one can find a match, and simply discards, there may be more than four. If someone makes a match, there may be less than four. If there is one card or zero cards on the table, a player cannot make a match and must discard one card.
5. Play proceeds with each person either making a match or discarding until they have no cards left in their hands. Then four more cards are dealt to each player, but no additional cards are put into the middle. A new person starts this second round by trying to make a match just as in the first round.
6. The game ends when all the cards in the deck have been used up. (On the last round of dealing, each player may not get four cards.) The winner is the person with the most cards in his or her "winnings" pile.

Sample Play. Player A has JH, 7D, 6S, 2S. Face-up cards are 8C, KC, 5H, 2D. Player A can make a match with 6S (+6) equal to 8C (+8) and 2D (-2). Player A could also match 7D (-7) with 5H (-5) and 2D (-2), but he/she cannot make both matches on one round.

Player B has QS, 9S, 3H, 1C. Face-up cards are KC and 5H. Although KC (+13) is equal to QS (+12) and 1C (+1), player B cannot use two cards from his or her hand to match one card on the board. Therefore, player B must discard one card from his or her hand.

Variation: 1. Let one card from a player's hand match more than two cards. For example, in the Sample Play above, player A could use 6S (+6) to match KC (+13), 5H (-5), and 2D (-2).

"Integer 21"

For two to six players

Skills. Mental addition of integers. Strategy.

Goal. To be the player closest to 21 points or -21 points.

Materials: A regular deck of cards. Let the red cards have negative values. Ace is 1 or 11 (-1 or -11).

Directions: 1. The dealer gives one card down and one card face up to each player.
2. Each player is given the opportunity to take

extra cards (face up) if she/he does not think the sum is close enough to 21 or -21 to beat all the other players (or all the players can play to beat the dealer). However, a player can take a *maximum* of four extra cards.

3. The winner is the person closest to 21 or to -21

Sample Play: Player A has KC, 3H (equal to +10) Takes another card, 7S. Now has +17. Doesn't want more cards

Player B has 9D, 8H (equal to -17). Does not take more cards. They would be tied as winners if no other player gets closer to 21 or -21 You can go over 21 or lower than -21—i.e., 23 beats 17—or you can have anyone who goes beyond those values go "bust"

Variation: 1 Use another number such as zero or 10 as the target number

"Maximo–500"

For two to four players

Skills: Operations with ets

Goal To make the largest possible value with your cards

Materials A deck of cards. Let the red cards have negative values. Paper to keep score on

Directions: 1 Each player is dealt three cards.

2 The players then try to make the largest possible value with any combination of adding, subtracting, multiplying, or dividing. For example

a 6H, 7S, 2D would produce (-6) X (+7) X (-2) = 84.

b 6S, 7H, 2C could only produce (+6) X (+2) X (-7) = 5, but 6 [2-(-7)] makes 54.

c. 6D, 7H, 2H would produce (-6 + -2) (-7) = 56.

3. Each player explains and records his or her score for the round, then is dealt three new cards. (Old cards are discarded.)

4. You can challenge a person if you think she/he did not compute correctly or did not find the maximum number of points. If you are right, you get his or her points. If you are wrong, the person you challenged gets your points.

5. After each round, each player adds his or her score to the previous total. The discards are reshuffled when the deck is used up.

6. The game ends when someone reaches a total of 500 or when a time limit is reached.

Variations. 1 Use a fraction or decimal deck

2 Use more than three cards

3 Play "Minimo". Try to make the least number of points. The first one to reach -500 is the winner.

4 Play to more or less than 500

PRIME FACTOR GAME

by Marion T. Carr

The following is an example of one of the many card games used in the general mathematics classes in West High School, Mankato, Minnesota. "Students play games, cards, and engage in activities that make class fun, hence students want to come to class." This was originally written as part of an article for the Saskatchewan Math Teachers Society, Fall 1977.

Cards and games should be fun and should reinforce a concept that has been taught. Most teachers find that students learn the rules for cards and games very rapidly, while having difficulty with simple, basic concepts in mathematics. The answer is that the student is motivated to learn the rules for cards because it is more relevant at this time than learning about mathematics.

A typical card game that reinforces a concept is the "Prime Factor Game." The deck consists of several factorable numbers and their prime factors. The teacher selects 15 or 20 factorable numbers and their prime factors. The numbers are placed on individual blank playing cards. Each card is made as shown in Figure 1.

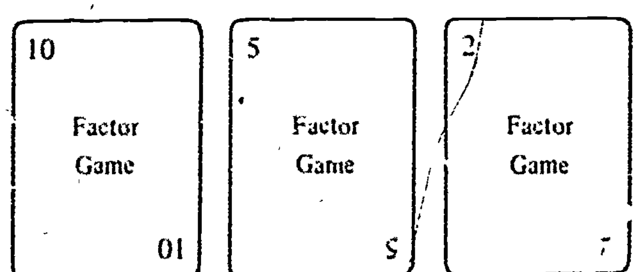


Figure 1

The object of the game is to obtain a book of cards consisting of a factorable number and its prime factors. An example would be Figure 1 - 10, 5, and 2. This book is then laid down on the table. The basic rules of "Rummy" are used. Seven cards are dealt to each player, the remaining cards are put into the center of the table face down, with one up as a discard or reserve pile. A player must discard, except on the final hand. The first player to lay down all his or her cards wins. Discarding is optional on the final hand. Each player plays in turn around the table, drawing, laying down, and/or discarding. ~~The winner gets a point for~~ each book and the losers a point against for each card left in their hands.

Variation: Make a deck with just factors of numbers.

Students need to achieve success. Cards, games, and puzzles are one of the ways to achieve this success. Kids, cards, and games can spell success.

VARIETY IS THE SPICE OF. . .A GAMEBOARD?

by Bettye C. Hall

Games, like manipulatives, are appropriate for students of all grade levels. The gameboard that follows can be used with skill cards to reinforce whatever concepts the teacher (or students) choose(s). The author is an Instructional Specialist with the Houston Independent School District.

Yes, a gameboard. One of the most successful activities to use with high school students is a game played on a gameboard that reinforces the concept(s) presented in class. Any set of problems that would normally ~~be presented in~~ drill sheet form can be used with the gameboard. The outstanding feature of the activity is its flexibility. Once the gameboard is constructed, it can be used over and over, only the skill cards are changed to alter the concept to reinforce.

Here is how it works. For every four students you need one gameboard, four game pieces, one die, and a set of skill cards. The gameboard and the skill cards can be made by the students, saving the teacher a great deal of time. The rules for playing the game are as follows. (1) the student draws a skill card, works the problem, and checks his or her solution with the solution on the back of the skill card, (2) if the solution is correct, she/he tosses the die and moves the number of spaces indicated if the solution is wrong, she/he loses his or her turn, (3) when a player lands on a space where there are additional directions written, she/he must follow those directions, (4) the first person to reach the "Winner" circle wins the game.

The skill cards are the heart of the activity. Each gameboard must have its own set of skill cards (30 to 40 are sufficient). The cards can be made so that all students are working on the same set of problems, or so that each group of four is working on the same concept but at different levels of difficulty. Each card has a question on one side and the answer on the back—e.g., a geometry term on the front and the definition on the back, a quadratic equation on the front and the roots on the back, an improper fraction on the front and the equivalent mixed number on the back. The possibilities are unlimited. If the unit being studied is factoring, each classroom set of cards could contain a specific type of factoring pattern—i.e., a set with trinomial squares, a set with the difference of two squares, a set with trinomials that are not squares, a set with polynomials that factor by grouping, and a set with the difference of two cubes. At the ~~end of~~ the unit a review set for factoring could be made by taking some cards from each of the sets above. A list of concepts for skill cards can be found at the end of this discussion.

The gameboard is designed on a letter-size file folder for easy storage. Stationery stores carry file folders in many colors. A variety of colors makes the gameboards more attractive. To construct a gameboard, choose a file folder and make the playing spaces using circular or rectangular stick-on labels. These need to be at least 2 to 3 centimetres wide so that alternate directions can be easily written inside the space. As you stick the labels on the file folder, make a winding path and have at least one loop as a detour path (see Figure 1). Use at least 30 labels the more you use, the longer the game. After all the labels are in place, begin marking the spaces with the alternate directions. If you use 35 circles and a detour similar to the illustration in Figure 1, the spaces can be marked as follows.

Space Number	Comment
1	Start
3	Move ahead one space
5	Detour (see Figure 2)
E	Take an extra turn
7	Lose one turn
10	Move back one space
13	Hop ahead two spaces
16	Go back four spaces
18	Take an extra turn
22	Skip ahead three spaces
27	Move back two spaces

Space Number	Comment
31	Jump ahead one space
33	Exact throw to win
35	Winner

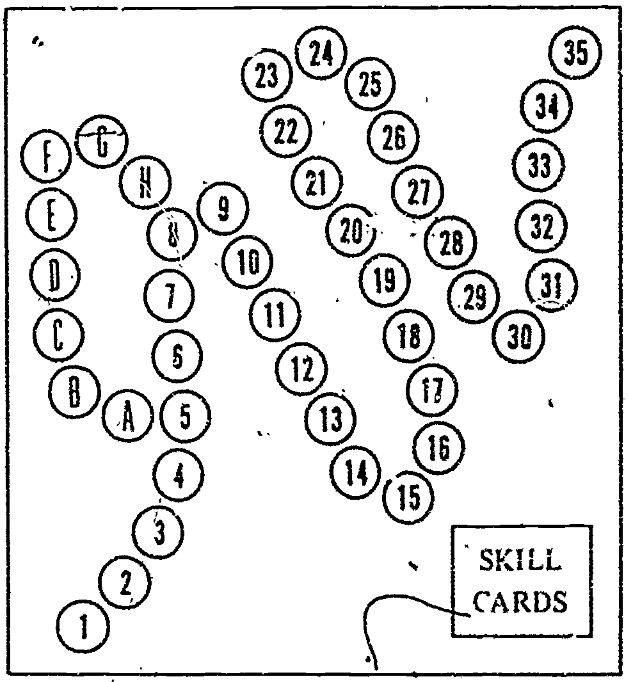


Figure 1

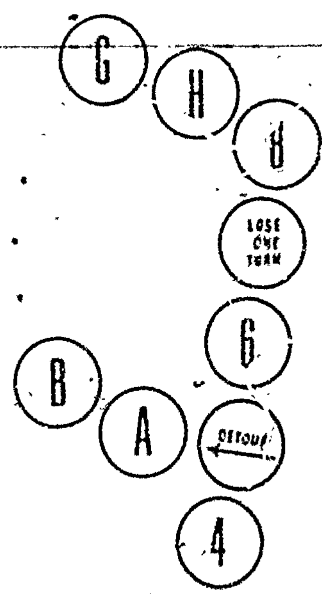


Figure 2

The spaces that do not contain alternate directions can be numbered or left blank. When writing alternate directions, be sure that they do not counteract each other e.g., "Move ahead two spaces" should not land you on a space marked "Lose one turn." Mark a space on the board for the skill cards (see Figure 1). To keep students from working ahead, the cards should be in an envelope and drawn from it as each student takes his or her turn.

If you wish to make the gameboard more interesting, use colorful stickers and clever comments matched to the stickers. Holiday stickers, flower stickers, fairy tale stickers, and the like can be used near the spaces where you have alternate directions. For example, use a hockey player near a space marked "Take another turn" and write under the hockey player "Outstanding assist to score the winning goal." Any type of sticker can be used. The sillier they are, the better the older students seem to like them (see Figure 3). All that is necessary is some appropriate comment to accompany the sticker. Generally 10 to 12 stickers will add enough color and spice to make the game more fun.

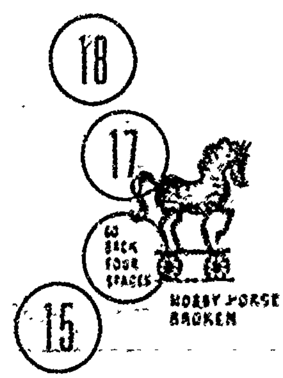


Figure 3

Print the rules to the game on the outside of the file folder. Keep the rules as simple as possible (see above). Remember that the object of the game is to have the students work the problems on the skill cards. Do not quibble with the students over the rules. Be sure that the rules they are using are fair to all players, and let them play the game. As long as they are doing the basic mathematics, algebra, geometry, or whatever, that is the important thing. Be sure to give your activity a name like "Rattdom Run," "The Great Chase," or "Keep On Moving." Ask the students for suggestions.

The possibilities for skill cards are limited only by your imagination. To get you started, here is a list of concepts I have used successfully. Do not limit your choices to these. They are only to get you started.

Front	Back
Factor $3x^2 + 5x - 2$	$(3x - 1)(x + 2)$
Find x $\log_4 64 = x$	$x = 3$
Find x $\text{Cot}\, x = -\sqrt{3}$	$x = 150°$ or $x = \frac{5\pi}{6}$
Simplify $\sqrt{27a^8 b^3}$	$3a^4b \sqrt{3b}$
Simplify $\tan\theta(1 - \sin^2\theta)$	$\sin\theta\cos\theta$
Solve $3x + 1 = x - 7$	$x = -4$

Front	Back
Solve $2x^2 - x = 1$	$x_1 = -\frac{1}{2}, x_2 = 1$
Evaluate $\frac{4a^2 + 4a}{3}$ for $a = -2$	$\frac{4}{3}$
Evaluate $\text{Cos}\, 60$	$\frac{1}{2}$
Give an equation for the line through (3,2) and parallel to $x + 4y = 7$	$(y - 2) = -\frac{1}{4}(x - 3)$ or $x + 4y = 11$